Exercice 2: (Au choix) (10 pts)

1-\ Si la Transformation suivante:

$$q = \sqrt{2P}\,\sin Q \;, \quad p = \sqrt{2P}\,\cos Q$$

est canonique, elle doit vérifier que $[q, p] = 1$

$$[q,p]_{Q,P} = \frac{\partial q}{\partial Q}\frac{\partial p}{\partial P} - \frac{\partial q}{\partial P}\frac{\partial p}{\partial Q} \qquad (1)$$

$$\frac{\partial q}{\partial Q} = \sqrt{2P}\,\cos Q \quad ; \quad \frac{\partial p}{\partial P} = \frac{1}{2}\frac{2\cos Q}{\sqrt{2P}} = \frac{\cos Q}{\sqrt{2P}} \qquad (1)+(1)$$

$$\frac{\partial q}{\partial P} = \frac{1}{2}\frac{2\sin Q}{\sqrt{2P}} = \frac{\sin Q}{\sqrt{2P}} \quad ; \quad \frac{\partial p}{\partial Q} = -\sqrt{2P}\,\sin Q. \qquad (1)+(1)$$

$$[q,p] = \left(\sqrt{2P}\cos Q\right)\left(\frac{\cos Q}{\sqrt{2P}}\right) - \left(\frac{\sin Q}{\sqrt{2P}}\right)\left(-\sqrt{2P}\,\sin Q\right) = \cos^2 Q + \sin^2 Q = 1.$$

2-\ Utilisons la Transf. pr résoudre le problème de l'oscillateur harmonique:

a) Le Hamiltonien de l'oscillateur harmonique linéaire:

$$H(q,p) = \frac{p^2}{2m} + \frac{k}{2}q^2.$$

b) puisque: $\frac{\partial G}{\partial t} = 0 \Rightarrow \mathcal{H} = H =$ avec $k = 1$ et $m = 1$., Ainsi

Ainsi $H = \frac{p^2}{2} + \frac{q^2}{2}$ (1)

$$\mathcal{H} = \frac{2P}{2}\cos^2 Q + \frac{1}{2}\,2P\sin^2 Q = P\left(\cos^2 Q + \sin^2 Q\right) \Rightarrow \mathcal{H} = P. \qquad (1)$$

c) Les éqs de Hamilton:

$$\dot{Q} = \frac{\partial \mathcal{H}}{\partial P} = 1 \Rightarrow \frac{dQ}{dt} = 1 \Rightarrow Q = \int dt \Rightarrow Q = t. \qquad (1)$$

$$\dot{P} = -\frac{\partial \mathcal{H}}{\partial Q} = 0 \Rightarrow P = c^{te} = E_T \qquad (0.5)+(0.5)$$

d) $q(t) = \sqrt{2E_T}\,\sin t$; $p(t) = \sqrt{2E_T}\,\cos t$. (0.5)+(0.5)