D'après l'éq. de Hamilton:

$$\dot{P}_\theta = -\frac{\partial H}{\partial \theta} = 0 \Rightarrow P_\theta = c^{te} = \frac{\partial W}{\partial \theta} = \alpha_1 \Rightarrow W = \int \alpha_1 \, d\theta \Rightarrow \boxed{W = \alpha_1 \theta} \quad (1)$$

(1)

$$\frac{(\alpha_1)^2}{2mR^2} + \frac{1}{2m}\left(\frac{\partial U}{\partial z}\right)^2 + \frac{k}{2}\left(R^2+z^2\right) = \alpha .$$

$$\Rightarrow \frac{\partial U}{\partial z} = \sqrt{2m\left[\alpha - \frac{k}{2}\left(R^2+z^2\right) - \frac{\alpha_1^2}{2mR^2}\right]}$$

$$\Rightarrow U(z) = \int \sqrt{2m\left[\alpha - \frac{k}{2}\left(R^2+z^2\right) - \frac{\alpha_1^2}{2mR^2}\right]} \, dz . \quad (1)$$

Ainsi: $S(\theta, z, t) = -\alpha t + \alpha_1 \theta + \int 2m\left[\alpha - \frac{k}{2}\left(R^2+z^2\right) - \frac{\alpha_1^2}{2mR^2}\right]$ (0.5)