Corrigeé -type 3ème Année

Exo 1 :

$$\Gamma(z) = \int_0^{\infty} e^{-x} \cdot x^{z-1}\, dx \quad ; \quad \beta(p,q) = \int_0^{1} t^{p-1} \cdot (1-t)^{q-1}\, dt$$

$$\beta(p,q) = 2\int_0^{\pi/2} \sin\theta^{2p-1} \cos\theta^{2q-1}\, d\theta \quad \text{et on a :} \quad \int_0^{1} t^{\alpha-1}(1-t^{\beta})^{\beta-1}\, dt = \frac{1}{\beta}\beta\left(\frac{\alpha}{\beta}, B\right)$$

- $I = \int_0^{\infty} e^{-x^2} \cdot x^9\, dx$ → changement de variable : $\begin{cases} x^2 = t \longrightarrow 2x\,dx = dt \rightarrow dx = \frac{dt}{2\sqrt{t}} \\ x = \sqrt{t} \longrightarrow x^9 = t^{9/2} \end{cases}$

$$I = \int_0^{\infty} e^{-t} \cdot t^{9/2} \cdot \frac{1}{2} t^{-1/2}\, dt = \frac{1}{2}\int_0^{\infty} e^{-t} \cdot t^{4}\, dt = \frac{1}{2}\Gamma(5) \qquad (2)$$

- $J = \int_0^{1} t^{1/2}(1-t^5)^{1/2}\, dt = ?$ On a : $\begin{cases} \alpha - 1 = \frac{1}{2} \\ \beta - 1 = \frac{1}{2} \\ p = 5 \end{cases} \Longrightarrow \begin{cases} \alpha = 3/2 \\ \beta = 3/2 \\ p = 5 \end{cases}$

$$J = \frac{1}{5}\beta\left(\frac{3/2}{5}, \frac{3}{2}\right) = \frac{1}{5}\beta\left(\frac{3}{10}, \frac{3}{2}\right) \qquad (2)$$

- $K = \int_0^{\pi/2} \sin^2\theta \cdot \sqrt{\tan\theta}\, d\theta = \int_0^{\pi/2} \sin^2\theta \cdot \sin\theta^{1/2} \cdot \cos\theta^{-1/2} \cdot \cos\theta\, d\theta$

$$= \int_0^{\pi/2} \sin\theta^{5/2} \cdot \cos\theta^{-1/2}\, d\theta$$

$$\begin{cases} 2p-1 = \frac{5}{2} \\ 2q-1 = -\frac{1}{2} \end{cases} \Longrightarrow \begin{cases} p = 7/4 \\ q = 1/4 \end{cases} \quad \text{donc} \quad K = \frac{1}{2}\beta\left(\frac{7}{4}, \frac{1}{4}\right) \qquad (2)$$