EXO2 :

Interpolation de Lagrange $P(x) = y_0 L_0 + y_1 L_1 + y_2 L_2$ (0,5)

les pts d'appuis : $\begin{cases} x_0 = 0 \\ x_1 = \frac{1}{6} \\ x_2 = \frac{1}{2} \end{cases} \Longrightarrow \begin{cases} y_0 = \sin 0\pi = 0 \\ y_1 = \sin \frac{\pi}{6} = \frac{1}{2} \\ y_2 = \sin \frac{\pi}{2} = 1 \end{cases}$ (0,5) (0,5) (0,5)

$y_0 = 0 \Longrightarrow y_0 L_0 = 0$; Calculons L_1 et L_2

$$L_1 = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{x(x-\frac{1}{2})}{\frac{1}{6}(\frac{1}{6}-\frac{1}{2})} = -18x^2 + 9x$$ (0,5) (0,5)

$$L_2 = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{x(x-\frac{1}{6})}{\frac{1}{2}(\frac{1}{2}-\frac{1}{6})} = 6x^2 - x$$ (0,5) (0,5)

$$P = \frac{1}{2}(-18x^2+9x) + 1(6x^2-x) = -3x^2 + \frac{7}{2}x$$ (1,5)

EXO3 :

$$\begin{cases} x' = 5x + y - z \\ y' = 2x + 4y - 2z \\ z' = x - y + 3z \end{cases} \Leftrightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} 5 & 1 & 1 \\ 2 & 4 & -2 \\ 1 & -1 & 3 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

chercher les Valeurs et les Vecteurs Propres de A.

$\det(A - \lambda I) = (x-6)(x-2)(x-4)$

$\lambda_1 = 2$ (0,5)

$\lambda_2 = 4$ (0,5)

$\lambda_3 = 6$