EX03

< à variable séparable >

1) $\frac{3y+1}{x-1} \cdot \frac{dy}{dx} = \frac{1}{xy} \iff (3y+1)\,xy\,dy = (x-1)\,dx$

$\iff (3y^2+y)\,dy = \frac{x-1}{x}\,dx = \left(1-\frac{1}{x}\right)dx$

$\iff \int (3y^2+y)\,dy = \int 1 - \frac{1}{x}\,dx$

$\iff y^3 + \frac{1}{2}y^2 = x - \ln x + C$

2

2) $xy' + y - x = 1 \iff xy' + y = x+1$

$(E) : y' + \frac{1}{x}y = \frac{x+1}{x}$

$(E_0) : y' + \frac{1}{x}y = 0 \iff y' = -\frac{1}{x}y$

$\iff \int \frac{y'}{y}dx = \int -\frac{1}{x}dx \implies \ln|y| = -\ln|x| + C = \ln|x|^{-1} + C$

$\iff \boxed{y_0 = \frac{k}{x}}$

1

$y_p = \frac{k(x)}{x} \longrightarrow y' = \frac{k'x - k}{x^2}$

$(E) \iff \frac{k'x-k}{x^2} + \frac{k}{x^2} = \frac{x+1}{x} \iff \frac{k'}{x} = \frac{x+1}{x} \implies k' = x+1 \implies k = \frac{x^2}{2} + x$

$\boxed{y_p = \frac{x}{2} + 1}$ 1 et donc $y_g = \frac{k}{x} + \frac{x}{2} + 1$

3) $y'' - 6y' + 8y = 2\sin \cancel{x}$

$(E_0) : y'' - 6y' + 8y = 0$

$(E.C) : r^2 - 6r + 8 = 0$

$\Delta = 4 \longrightarrow r_1 = 2$ et $r_2 = 4$

$\boxed{y_0 = \lambda_1 e^{2x} + \lambda_2 e^{4x}}$

1

avec second membre

$\begin{cases} \lambda_1' y_1 + \lambda_2' y_2 = 0 \\ \lambda_1' y_1' + \lambda_2' y_2' = \sin x \end{cases}$

$y_1 = e^{2x} \longrightarrow y_1' = 2e^{2x}$

$y_2 = e^{4x} \longrightarrow y_2' = 4e^{4x}$

$\begin{cases} \lambda_1' e^{2x} + \lambda_2' e^{4x} = 0 \\ 2\lambda_1' e^{2x} + 4\lambda_2' e^{4x} = 2\sin x \end{cases}$

0,5