$\lambda_1 = 2 \longrightarrow (A - 2I)V_1 = 0 \iff \begin{pmatrix} x=0 \\ y=z \end{pmatrix}$ (0.5)

$V_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ (1)

$\lambda_2 = 4 \longrightarrow (A - 4I)V_2 = 0 \iff \begin{cases} x=z \\ y=0 \end{cases}$ (0.5)

$V_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ (1)

$\lambda_3 = 6 \longrightarrow \begin{cases} x=y \\ z=0 \end{cases}$ (0.5)

$V_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (1)

La solution du système : $\begin{cases} x(t) = \alpha_3 e^{6t} + \alpha_2 e^{4t} & (0.5) \\ y(t) = \alpha_1 e^{2t} + \alpha_3 e^{6t} & (0.5) \\ z(t) = \alpha_1 e^{2t} + \alpha_2 e^{4t} & (0.5) \end{cases}$

La solution est encore donnée par

$$\vec{X}(t) = \alpha_1 e^{\lambda_1 t} V_1 + \alpha_2 e^{\lambda_2 t} V_2 + \alpha_3 e^{\lambda_3 t} V_3$$

$$X(t) = \alpha_1 e^{2t} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \alpha_2 e^{4t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \alpha_3 e^{6t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad (1)$$