$$\begin{cases} \lambda_1' e^{2x} + \lambda_2' e^{4x} = 0 \quad \times\ e^{-2x} \\ 2\lambda_1' e^{2x} + 4\lambda_2' e^{4x} = 2\sin x \end{cases} \Rightarrow \begin{cases} \lambda_1' + \lambda_2' e^{2x} = 0 \\ 2(-\lambda_2' e^{2x}) e^{2x} + 4\lambda_2' e^{4x} = 2\sin x \end{cases} \Rightarrow \begin{cases} \lambda_1' = -\lambda_2' e^{2x} \\ \text{---} \quad (2) \end{cases}$$

$$(2) \Leftrightarrow 2\lambda_2' e^{4x} = 2\sin x$$

$$\Leftrightarrow \lambda_2' = e^{-4x}\sin x$$

$$\Rightarrow \lambda_2 = \int e^{-4x}\sin x\, dx \quad \text{----} \quad \text{Intégrale par partie}$$

$$\begin{cases} u = \sin x \longrightarrow u' = \cos x \\ v' = e^{-4x} \longrightarrow v = -\frac{1}{4} e^{-4x} \end{cases}$$

$$\lambda_2 = \frac{-e^{-4x}}{4}\sin x + \frac{1}{4}\underbrace{\int e^{-4x}\cos x\, dx}_{\alpha}$$

Calculons α par partie

$$u = \cos x \longrightarrow u' = -\sin x$$

$$v' = e^{-4x} \longrightarrow v = -\frac{1}{4}e^{-4x}$$

$$\alpha = -\frac{e^{-4x}}{4}\cos x - \frac{1}{4}\int \sin x . e^{-4x}\, dx$$

On a donc :

$$\lambda_2 = \int e^{-4x}\sin x\, dx = \frac{-e^{-4x}}{4}\left(\sin x + \frac{\cos x}{4}\right) - \frac{1}{16}\int \sin x . e^{-4x}\, dx$$

$$\text{donc} \quad \underbrace{\frac{17}{16}\int e^{-4x}\sin x\, dx}_{} = \frac{-e^{-4x}}{4}\left(\sin x + \frac{\cos x}{4}\right) = \frac{-e^{-4x}}{16}(4\sin x + \cos x)$$

$$\boxed{\lambda_2 = \frac{-1}{17} e^{-4x}\left(4\sin x + \cos x\right) + k_2}$$

1,5

$$\lambda_1' = \frac{1}{17} e^{-2x}\left(4\sin x + \cos x\right) \longrightarrow \text{Par la m̂ démarche On aura}$$

$$\lambda_1 = \frac{e^{-2x}}{5}(\cos x + 2\sin x) + k_1$$

1,5

$$y_g = \frac{\cos x + 2\sin x}{5} + k_1 e^{2x} - \frac{1}{17}(4\sin x + \cos x) + k_2 e^{4x}$$

0