Ex : 2

On prends $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$|\psi_+\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle + |1\rangle \right) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \quad 0{,}5$$

$$|\psi_-\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) = \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \quad 0{,}5$$

- l'ensemble (a) le système est dans l'état $|\psi_+\rangle$

$$\hat{\rho}_A = |\psi_+\rangle\langle\psi_+| = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad 1$$

- L'ensemble (b) le système est dans l'état $|\psi_-\rangle$

$$\hat{\rho}_B = |\psi_-\rangle\langle\psi_-| = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \quad 1$$

- L'ensemble (c) le système est 50% dans $|\psi_+\rangle$ et 50% dans $|\psi_-\rangle$

$\hat{\rho}_+ = \hat{\rho}_A$; $\hat{\rho}_- = \hat{\rho}_B$ et $\hat{\rho}_C = \sum_i p_i \hat{\rho}_i$

$$\hat{\rho}_C = \frac{1}{2}\hat{\rho}_+ + \frac{1}{2}\hat{\rho}_-$$

$$1 \quad = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

2) $\hat{\rho}_A^2 = \left(\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right)^2 = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \hat{\rho}_A \longrightarrow \quad 1$

$\hat{\rho}_B^2 = \left(\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right)^2 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \hat{\rho}_B \quad 1$

$\hat{\rho}_C^2 = \left(\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)^2 = \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \neq \hat{\rho}_C \quad 1$

- L'ensemble (A) $\langle\sigma_x\rangle = \mathrm{Tr}\left(\hat{\rho}_A \sigma_x\right) = 1$; $\langle\sigma_y\rangle = \langle\sigma_z\rangle = 0 \quad 1$

" (B) $\langle\sigma_x\rangle = -1$; $\langle\sigma_y\rangle = \langle\sigma_z\rangle = 0 \quad 1$

" (C) $\langle\sigma_x\rangle = \langle\sigma_y\rangle = \langle\sigma_z\rangle = 0 \quad 1$