Ex : 3

- $|A\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right]$

$$= \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$$

$$\rho_A = |A\rangle\langle A| = \begin{pmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}\left(1/\sqrt{2}, 1/\sqrt{2}\right)$$

$$= \begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$$ 02

$$\langle \hat{S}_x\rangle = \mathrm{Tr}\left(\hat{\rho}_A \cdot \hat{S}_x\right) = \mathrm{Tr}\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}\cdot\begin{pmatrix}0 & \hbar/2\\\hbar/2 & 0\end{pmatrix}$$

$$= \mathrm{Tr}\begin{pmatrix}\hbar/4 & \hbar/4\\\hbar/4 & \hbar/4\end{pmatrix}$$

$$= \hbar/2$$

de même $\langle S_y\rangle = 0$ et $\langle S_z\rangle = 0$ 01,5

- Le système (B) $\hat{\rho}_B = \frac{1}{2}\left(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|\right)$

$$= \frac{1}{2}\left(\begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \begin{pmatrix}0\\1\end{pmatrix}(0\ 1)\right)$$

$$= \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}$$ 02

On trouve $\langle S_x\rangle = \langle S_y\rangle = \langle S_z\rangle = 0$ 0,5

- $\hat{\rho}_A^2 = \begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}^2 = \begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix} = \hat{\rho}_A \longrightarrow$ l'état $|A\rangle$ est pur. 0,5

$\hat{\rho}_B^2 = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}^2 = \frac{1}{2}\hat{\rho}_B \longrightarrow$ l'état $|B\rangle$ est mixte. 0,5