Market Models and Nash Equilibrium Calculation in a Deregulated Electricity Market

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***Abstract*— in this paper we present the two most common models of electricity market: the bilateral Forward market model and the centralized spot market model. And In order to calculate the market equilibrium, of both models a competitive coevolutionary algorithm is used. The agents are modeled as a population of strategies that are led by an evolutionary algorithm toward the equilibrium point where each agent maximizes his profit.**

***Index Terms*-- Electricity Market; Forward Market; Spot Market; Nash Equilibrium; Competitive Coevolutionary Algorithm.**

I. Introduction

Since the deregulation of electricity market has been initiated, different market structures have appeared. The anatomy of deregulated power markets worldwide shows that the reform process has taken a number of different forms in various countries. Economic and political reasons, due to local conditions, have led to the adoption of different paradigms by the newly established market

Despite these differences, the competitive aspect of the generation sector is the most common basis in deregulated electricity markets models. Another common characteristic to deregulated electricity market is the oligopolistic aspect of the market. Few competing firms act strategically to maximize their profit. Unlike, perfect competition or monopoly markets where agents' interactions can be dealt with as a simple optimization problem, agents' strategic interactions in an oligopoly market need a game theoretic approach.

Our aim in this paper is to model the forward and spot transaction in the electricity market, and to use a coevolutionary algorithm in order to calculate the market equilibrium in each market model.

The rest of the paper is presented as follow; in the next section the game theory and the notion of Nash equilibrium are presented. Section III presents forward and spot transactions models of the electricity market. In section IV we develop a competitive coevolution approach to the market Nash equilibrium calculation.

II. Game Theory and Nash Equilibrium

Agents' strategic interactions and their strategic behavior are commonly studied by a game theoretic approach. Game theory attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others. In game theory, a game consists of a set of players, a set of strategies available to those players, and a specification of payoffs for each combination of strategies, and the traditional applications of game theory attempt to find equilibria in those games.

In game theory, Nash equilibrium [1] is a solution concept of a non cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy unilaterally. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep their unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.

A set of strategies is Nash equilibrium if no player can do better by unilaterally changing his strategy. Let $\left(S,\pi\right)$ be a game with n players, where S_i the strategy set for player is i, $S = S_1 \times S_2 \ldots \times S_n$ is the set of strategy profiles and $\pi = \left\{\pi_1(s),\ldots,\pi_n(s)\right\}$ is the payoff function. Let x_{-i} be a strategy profile of all players except for player i. When each player $i \in \left\{1,\ldots,n\right\}$ chooses strategy s_i resulting in strategy profile $s = \left(s_1,\ldots,s_n\right)$ then player i obtains payoff $\pi_i(s)$. Note that the payoff depends on the strategy profile chosen, i.e. on the strategy chosen by player i as well as the strategies chosen by all the other players. A strategy profile $s^* \in S$ is a Nash Equilibrium (NE) if no unilateral deviation in strategy by any single player is profitable for that player, that is

$$\forall i, s_i \in S_i, s_i \neq s_i^* : \Pi_i(s_i^*, s_{-i}^*) \geq \Pi_i(s_i, s_{-i}^*) \qquad (1)$$

Game-theory based methods are widely used to model generators’ strategic interactions and to search the Nash market equilibrium.

III. THE ELECTRICITY MARKET MODELING

The role of electricity market is to establish a fair trading platform for exchange of electrical energy between suppliers (generating companies), consumers (retailers, distributers) and other financial entities, for both short and long-term. Typically, market agents can take part on different transactions [2][3].

- In the forward market transactions are made several months prior to the delivery. Forward transactions are purely financial and their concretization will occur in the real time. They are not conditioned by production capacity or transmission limitations.
- The spot market takes place few hours prior to the delivery (day-ahead, hour-ahead or real time). Transactions in the spot market are very critical to the stability and security of the power system, and have to be according to the power system limitations and constraints.

Several entities participate to the transactions, the most important are:

- Suppliers: production companies, neighbor systems or intermediaries can participate as suppliers and try to sell their energy in the market.
- Consumers: cities, distribution companies or intermediaries can participate to the transactions and buy energy from the market of from the suppliers directly.
- Independent System Operator: the ISO is an independent and non profitable organism. The ISO has to ensure a reliable and secure functioning of the power system and to maximize the social welfare from the market transactions.

A. Electricity markets models

In this paper, we are mainly interested by the strategic behavior of the suppliers in the different markets. The suppliers' models and decisions making processes in both forward and spot markets are also presented.

Bilateral forward market

In the bilateral forward market, suppliers and consumers directly negotiate the price and the quantity of traded energy several months prior to the delivery. In this paper, the bilateral market is analyzed using Cournot model, suppliers compete in term of quantities to be sold, and consumers are defined by their demand function where the amount of energy to be acquired is inversely proportional to the price [2].

Consumers' forward demand functions are of the form:

$$D_i^f(p_i^f) = D_i^0 - e_i p_i^f \qquad j = 1,\ldots,N_d \tag{2}$$

The nodal marginal clearing price is defined as follow:

$$p_i^f = \frac{1}{e_i}(D_i^0 - \sum_{i=1}^{N_d} q_{ij}) \qquad j = 1,\ldots,N_d \tag{3}$$

Producers' profit in the forward market is:

$$\Pi_i^f = \sum_{j=1}^{N_d} q_{ij} p_i^f - Cst_i(\sum_{j=1}^{N_d} q_{ij}) \tag{4}$$

Let us assume:

$$q_i^f = \left\{ q_{i1}^f, q_{i2}^f, \ldots q_{i_{N_d}}^f \right\}$$

$$\begin{aligned} p_i^f &= \frac{q_{ij}}{\sum q_i^f} p_j^f \\ i &= 1,\ldots,N_g \\ j &= 1,\ldots,N_d \end{aligned} \tag{5}$$

So the profit of a supplier from forward transaction can be formulated as:

$$\Pi_i^f\left(q^f\right) = q_i^f \bullet p_i^f - Cst_i\left(q_i^f\right) \qquad i = 1,\ldots,N_g \tag{6}$$

Where $\bullet$ denote the scalar product of vector q_i^f and p_i^f

Spot Market Model

The spot market is organized as a centralized market with maximalist ISO and the trading is done few hours prior to the delivery. In the centralized market, there is no direct trading between suppliers and consumers. The ISO acts as single buyer, it collects bids for power to purchase from the consumers and decides on quantity to be purchased from suppliers.

The spot market is analyzed using supply function model function [4,5,6,7]. We assume that:

- Consumers are defines by their demand D_j^s where $j = 1,\ldots,N_d$.
- Suppliers are defined by their cost: $Cst_i = 0.5\alpha_i q_i^2 + \beta_i q_i$ with $i = 1,\ldots,N_g$
- A linear supply function model of the form:

$$\begin{aligned} p_i^s &= \beta_i + x_i q_i^s \\ q_i^{min} &\le q_i^s \le q_i^{max} \\ i &= 1,\ldots,N_g \end{aligned} \tag{7}$$

With β is the parameter from the cost function.

In order to ensure the economical and the physical reliability of the power system, the ISO performs an OPF

routine to calculate the dispatched prices p_i^s and quantities q_i^s for each supplier. In this work, we use a Quadratic DC-OPF to calculate the dispatched price and quantities considering the suppliers bids [8]. The DC-OPF is formulated as follow:

$$\left[q_i^s, p_i^s\right] = \arg\min\left(\sum_{i=1}^{N_d} \frac{1}{2} x_i q_i^{s\,2} + \beta q_i^s\right) \tag{8}$$

Subject to:

$$\begin{aligned} A_{eq} q_i^s &= B_{eq} \\ A_{ineq} q_i^s &\leq B_{ineq} \\ q_i^{min} \leq \mathrm{q}_i^s &\leq \mathrm{q}_i^{max} \\ i &= 1,\ldots,N_g \end{aligned} \tag{9}$$

Thus the supplier's profit in the spot market will be expressed as:

$$\Pi_i^s(x) = p_i^s\left(q_i^s - \sum q_i^f\right) - Cst_i\left(q_i^s\right) \tag{10}$$

B. Market equilibrium calculation

To formulate the market equilibrium problem, we consider that:

- Suppliers act strategically by choosing the quantity of power to be sold to consumers in order to maximize their profit.
- Consumers have no strategic behavior: they can only affect the forward market price by their inverse demand function.
- The ISO is primarily concerned by the preservation of system constraints and has to rejects any transaction affecting the system integrity.
- System constraints in equation (9) are introduced as coupled constraints in the market equilibrium problem. [8,9,10].

Let us assume:

- q_i^{f*} and x_i^* are the Nash equilibrium strategies of the agent i
- q_{-i}^{f*} and x_{-i}^* are the Nash equilibrium strategies of all players except agent i

Strategies q_i^{f*} are the Nash equilibrium strategies of the forward market if and only if:

$$\left({}^{*} \quad {}^{*} \right) \qquad {}_{*}\left(\qquad {}^{*} \right)$$

generation to stay in population; if it fails, it is eliminated under the pressure of selection. Eliminated strategies can become optimal again. Loosing such strategies can make the progress toward optimality no longer guaranteed [16].

To ensure convergence toward optimal solution, individuals selected to be part of next generation have to be capable of defeating all the prior individuals. If this is not ensured, the algorithm can get stuck in weak strategies. To counter that, we seek an algorithm that progresses by producing new strategies that defeat older ones, and avoids the loss of interesting strategies. In order to achieve this goal we use a special instance of Evolutionary Algorithms: Evolution Strategies (ES) with crowding technique to preserve population diversity.

Evolution Strategies

Evolution Strategies Algorithm (ES) is one of the main paradigms in Evolutionary Computation [17,18], unlike Genetic Algorithms, ES is primary concerned by the mechanism of mutation, and the selection is based on the behavior (phenotype) of individuals, and are not concerned by the scheme theory as the GA are, unlike GA, mutation is the primary evolution operator, the representation is real coded and selection is mainly deterministic.

In a simple ES Algorithm noted (μ, λ) ES, there are μ parents used to create λ descendant by mutation, the μ best individuals of the μ parent and λ descendants are selected to be the new generation, this process is repeated for γ generations.

Algorithmic

In this work we use a crowding technique instead of the standard deterministic selection used in ES. Each parent is compared with his offspring and the best individual is selected to be part of the new generation. The evaluation is done using a random set of opponents' strategies,

The used algorithm is as follow:

```
for i=1:N_pop
   for j=1:M_indiv
      choose S %
%S is a Set of N random strategies from
%other populations
      P_fitness=Evaluate(parent, S);
      for k=1:L
         child  =Mutation(parent);
         C_fitness=Evaluate(child, S);
         if c_fitness>=P_fitness
            parent = child;
         end
      end
   end
end
```

This procedure is repeated for a predefined number of generations, at the end the best strategy of each population is selected.

V. CASES STUDY

To test the effectiveness of the proposed algorithm in the case of electricity market equilibrium, the IEEE 30 bus system is used. The system data are taken from the MATPOWER 3.5 data file 'case30pwl', with 6 generators and 13 loads. Table 1 shows the cost function parameters of the 6 suppliers, and table 2 depicts the load at each bus.

We assume there are 6 Suppliers and 5 consumers in the electricity market. Each supplier has a generation unit, and a consumer has several loads as depicted in the table 3. The demand functions of the consumers are presented in table 4.

Supplier	Bus	α [\$/MWh^2]	β [\$/MWh]
1	1	0.15	25
2	2	0.25	20
3	13	0.2	23
4	22	0.25	22
5	23	0.2	20
6	27	0.15	22

Table 1 suppliers cost function

Pd [MWh]	Bus	Pd [MWh]	Bus
43.40	2	18.00	17
4.80	3	6.40	18
15.20	4	19.00	19
188.40	5	4.40	20
45.60	7	35.00	21
60.00	8	6.40	23
11.60	10	17.40	24
22.40	12	7.00	26
12.40	14	4.80	29
16.40	15	21.20	30
7.00	16		

Table 2 System Load

Consumer	**Connected Loads**
1	1; 2; 3
2	4; 5
3	6
4	7; 8; 9; 10; 11; 12; 13; 14; 15; 16; 17; 18;
5	19; 20; 21

Table 3 Connected loads in each consumer

consumers	D_0 [MWh]	e [MWh/\$]
1	63.40	0.50
2	234.00	3.00
3	60.00	1.00
4	176.40	2.00
5	33.00	0.50

Table 4 consumers demand function

We calculate the Nash equilibrium point for both the forward and the spot market separately, tables 6 and 7 reports the obtained Nash equilibrium strategies, the contracted quantities and prices in both forward and spot contracts.

	q_{ij}^{f} [MWh]				
Suppliers	**1**	**2**	**3**	**4**	**5**
1	5.37	18.96	3.14	14.64	1.26
2	5.82	19.29	3.83	14.90	1.15
3	4.98	18.84	3.07	13.94	0.94
4	5.17	17.74	2.23	13.17	0.62
5	6.33	22.10	4.23	16.80	2.30
6	6.63	23.73	4.84	17.26	2.06

Table 5 Forward Equilibrium strategies

Suppliers	q_i^f **[MWh]**	p_i^f **[\$/MWh]**
1	43.37	37.62
2	44.99	37.60
3	41.77	37.64
4	38.93	37.75
5	51.76	37.51
6	54.52	37.51

Table 6 prices and quantities in the Forward market

Supplier	x_i^* **[\$/MWh²]**	q_i^s **[MWh]**	p_i^s **[\$/MWh]**	Π_i **[\$]**
1	0.2103	89.84	43.89	1092.00
2	0.3011	79.37	43.90	1109.30
3	0.2475	82.75	43.88	1042.90
4	0.3296	67.45	43.87	906.70
5	0.2431	92.11	43.91	1354.10
6	0.2264	98.60	43.93	1432.70

Table 7 Spot equilibrium strategies

We can notice that the prices in the spot market are higher than those in the forward market. Due to the demand elasticity of the consumers, the outcome of the forward market can be influenced.

In the spot market which is close to real time operation, consumers have to acquire the needed power whatever the price. This behavior allow the suppliers to rise their price far beyond their marginal prices (table(1)).

VI. CONCLUSIONS

This paper presents different models of the electricity market, and an approach to calculate the market equilibrium in each model. In the proposed approach each agent is modeled as a population of strategies and a variant of an Evolution Strategies algorithm is used as a learning method to find out the best strategies to be used.

a new approach to find out the market equilibrium in deregulated electricity markets modeled as a two stages stochastic game. A competitive coevolutionary algorithm is used to model the interactions of market agents in the market. Each agent is represented by a population of strategies and uses Nested Evolution Strategies as learning method to find better behaviors to overcome opponents' strategies, and thus, find their Nash equilibrium strategies.

To validate the proposed algorithm, an IEEE 30 bus test system is used, and three cases studies are carried out. In the different cases considered, the proposed algorithm has been successfully applied to calculate the Nash equilibrium and to the analysis of the strategic behavior of market agents in both forward and spot transactions.

The obtained results show the strategic behavior of market agents in the electricity market. The suppliers can exploit the structural weaknesses of the market and the limitations of the electricity network in order to increase the market prices and thus improve their payoff.

The key feature of our approach is the combination of a powerful learning algorithm to find the optimal strategies, and a scenario formulation to model the market uncertainties through a finite number of scenarios. The main advantage of this approach is the possibility to model different and realistic market models, and the simplicity and the efficiency in dealing with the system constraints even in the presence of uncertainties.

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