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Existence of solutions for solitons type equations in several space dimensions : Derrick's Problem with (r, p)-Laplacian

Abdelkader Dellal[1], Johnny Henderson[2] and Abdelghani Ouahab[3]

[1]Department of Mathematics and Computer Science,
Ahmed Ben Yahia El Wancharissi University.
Tissemsilt, Algeria.
E-mail: adellal@yahoo.fr

[2]Department of Mathematics
Baylor University
Waco, Texas 76798-7328 USA
E-mail: Johnny_Henderson@baylor.edu

[3]Laboratory of Mathematics
University of Sidi Bel Abbès
P.O. Box 89, 22000 Sidi-Bel-Abbès, Algeria
E-mail: agh_ouahab@yahoo.fr

Abstract

In this paper we study a class of Lorentz invariant nonlinear field equations in several space dimensions. The main purpose is to obtain soliton-like solutions with twice (r, p)-Laplacian. The fields are characterized by a topological invariant, which we call the charge. We prove the existence of a static solution which minimizes the energy among the configurations with nontrivial charge.

Key words and phrases: Soliton, variational calculus, splitting lemma.

1 Introduction

A soliton is a solution of a field equation whose energy travels as a localized packet and which preserves its form under perturbations. In this respect solitons have a particle-like behavior and they occur in many areas of mathematical physics, such as classical and quantum field theory, nonlinear optics, fluid mechanics, and plasma physics; see [8]. Probably, the simplest equation which has soliton solutions is the sine-Gordon equation,

$$-\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial t^2} + \sin\psi = 0, \tag{1.1}$$

where $\psi = \psi(x,t)$ is a scalar field, x, t are real numbers representing, respectively, the space and the time variable. Derrick, in a celebrated paper [7], considers the more realistic three-space-dimension model,

$$-\Delta\psi + \frac{\partial^2\psi}{\partial t^2} + V'(\psi) = 0, \tag{1.2}$$

Δ being the 3-dimensional Laplace operator and V' is the gradient of a nonnegative C^1 real function V. In [7] it is proved by a simple rescaling argument that (1.2) does not possess any nontrivial finite-energy static solution. This fact leads the author to say, "We are thus faced with the disconcerting fact that no equation of type (1.2) has any time-independent solutions which could reasonably be interpreted as elementary particles." Derrick proposed some possible ways out of this difficulty. The first proposal was to consider models which are the Euler-Lagrange equations of the action functional relative to the functional

$$S = \iint \mathcal{L} dx dt,$$

where the Lorentz invariant Lagrangian density proposed in [7] has the form

$$\mathcal{L}(\psi) = -\left(|\nabla\psi|^2 - |\psi_t|^2\right)^{\frac{p}{2}} - V(\psi), \quad p > 3. \tag{1.3}$$

However, Derrick does not continue his analysis and he concludes that a Lagrangian density of type (1.3) leads to a very complicated differential equation. He has been unable to demonstrate either the existence or nonexistence of stable solutions. In this spirit, a considerable amount of work has been done by Benci and collaborators, and a model equation proposed in [2]. The Lorentz invariant Lagrangian density proposed in [2] has the form

$$\rho = |\nabla\psi|^2 - |\psi_t|^2; \ \alpha(\rho) = a\rho + b|\rho|^{\frac{p}{2}}, \ p > n,$$

$$\mathcal{L}(\psi, \rho) = -\frac{1}{2}\alpha(\rho) - V(\psi). \tag{1.4}$$

In the case where p is constant, various mathematical results (existence, multiplicity results, asymptotic behavior, ...), have been obtained for different classes of solution models (see [1–6, 8, 9] and the references therein).

The aim of this study is to carry out an existence analysis of the finite-energy static solutions in more than one space dimension for a class of Lagrangian densities $\mathcal{L}$ which include (1.4) with (r,p)-Laplacian.

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