Séminaire de Mathématiques et Informatique

Université Djilali Liabès - Sidi Bel Abbès - le 18 novembre 2023

Topological structures and analysis in quasi-metric spaces

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Abstract

The objective of this presentation is to introduce the concept of $b-$metric space, also known as quasi-metric spaces. Within the scope of this study, we give topological structure inherent to this category of quasi-metric spaces. Furthermore, we explore classical theorems, including those pioneered by Cantor, Banach, Baire, and Stone. Additionally, our investigation extends to the b-metrization of the Heisenberg group and the b-ultrametrization of Gromov boundary space.

Subject Class[2010] : 47H10, 47H40, 54H25

keywords : Quasi-metric, fixed point, Heisenberg group, optimal transport, Hyperbolic space, Gromov-boundary.

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