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Three dimensional almost contact metric manifolds with a new perspective

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Abstract

The geometrical objects, functions, vector fields, 1-forms, and, in general, tensors on any manifold have an important role in differential geometry, especially in the construction of structures on manifolds.

The notion of almost contact structure was introduced by Boothby and Wang [2]. These manifolds were studied as an odd dimensional counterpart of almost complex manifolds and warped product are used to give examples of almost contact metric manifolds.

In [3], D. Chinea and C. Gonzalez have obtained a classification of the almost contact metric manifolds, studying the space that possesses the same symmetries as the covariant derivative of the fundamental 2-form. This space is decomposed into twelve irreducible components $C_1, ..., C_{12}$.

In dimension 3, the classes C_i reduce to the classes: C_0 class of cosymplectic manifolds, C_5 class of β-Kenmotsu manifolds, C_6 class of α-Sasakian manifolds, C_9-manifolds and C_{12}-manifolds.

Many works have focused on three-dimensional almost contact metric structures, either by studying and classifying them or by employing them to study various geometric topics.

Recently, in [1], the author has presented an interesting expression that generalizes the five classes of 3-dimensional almost contact metric structures and introduced a general approach to classify invariant almost contact metric structures on 3-dimensional Lie algebras.

Our aim in this talk is to construct all almost contact metric structures on a 3-dimensional Riemannian manifold. We define an interesting method to construct them starting from a unit vector field ξ. That's why ξ is called the characteristic vector field or the structure vector field. By this approach, we discuss the nature of 3-dimensional almost contact metric manifolds. Moreover, these techniques have made the classification simpler and more transparent.

Mathematics Subject Classification : 53C15, 53C25, 53Dx.

keywords : Almost contact metric structures, 3-dimensional manifolds, Lie algebras, Vector fields, Riemannian manifolds, Geodesic fields, Warped products, Invariant structures.

References

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