

EXISTENCE THEORY FOR IMPULSIVE PARTIAL HYPERBOLIC DIFFERENTIAL EQUATIONS OF FRACTIONAL ORDER AT VARIABLE TIMES

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Abstract. In this paper, we investigate the existence and uniqueness of solutions of a class of partial hyperbolic differential equations with impulses at variable times involving the Caputo fractional derivative. Our results are based on suitable fixed point theorems.

Key Words and Phrases: Impulsive hyperbolic differential equations, fractional order, solution, left-sided mixed Riemann-Liouville integral, Caputo fractional-order derivative, variable times, fixed point.

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