

Probability

Rules of Probability Calculations

Benchikh Tawfik

Faculty of Medicine
1st Year Medicine

November 23, 2025



Plan de cours

- 1 Rules of Probability Calculations
 - Rule of Complementary Probabilities
 - Complete System of Events
 - Conditional Probabilities
 - Conditional Probabilities-Examples
 - Law of Total Probability
 - Bayes' Theorem
 - Independent Events
- 2 Exercises



Rule of Complementary Probabilities

- The probabilities of an event E and its complement \bar{E} always add up to 1:

$$E \cup \bar{E} = \Omega$$

Hence,

$$\begin{aligned}\Pr(\Omega) = 1 &= \Pr(E \cup \bar{E}) \\ 1 &= \Pr(E) + \Pr(\bar{E})\end{aligned}$$

Rule of Complementary Probabilities: Example

- Roll a die. What is the probability of obtaining a number less than 6?

◇ E : "obtaining a number less than 6" $\Rightarrow \bar{E}$: "obtaining 6"

$$\Pr(\bar{E}) = q = \frac{1}{6} \implies \Pr(E) = p = 1 - \Pr(\bar{E}) = \frac{5}{6}.$$

- In a deck of 52 cards, 13 cards are drawn at random. What is the probability that at least one of them is an ace?

◇ E : "there is at least one ace" $\Rightarrow \bar{E}$: "there is no ace".

◇ Therefore,

$$P(\bar{E}) = q = \frac{C_{48}^{13}}{C_{52}^{13}} = \frac{6327}{20875} \implies p = 1 - q \approx 69.6\%.$$

Complete System of Events

- Let E be an event that can occur in several mutually exclusive ways, that is:

$$E = \bigcup_{i=1}^n E_i \quad \text{and} \quad E_i \cap E_j = \emptyset \quad \text{for } i \neq j.$$

(This is called a **complete system of events**.)

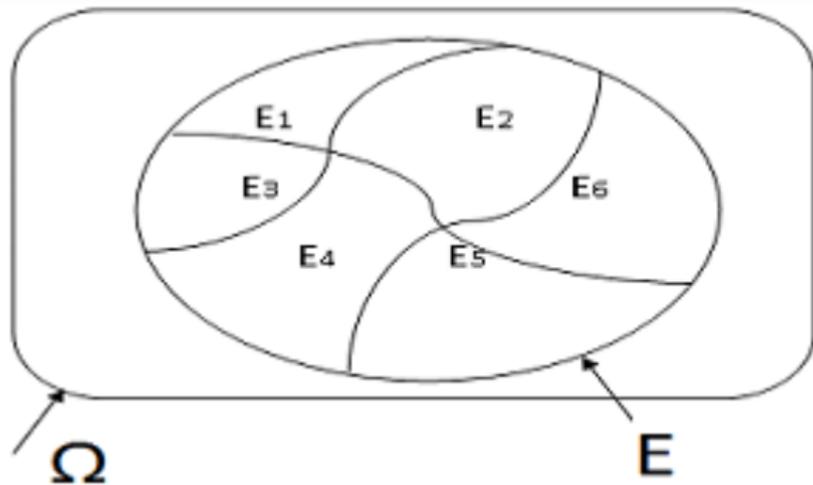


Figure: Complete system of events

Complete System of Events

- Then, the total probability of E is the sum of the probabilities of the disjoint events that compose it:

$$\begin{aligned}\Pr(E) &= \Pr(E_1 \cup E_2 \cup \dots \cup E_n) \\ &= \Pr(E_1) + \Pr(E_2) + \dots + \Pr(E_n)\end{aligned}$$

Complete System of Events: Example

- Two dice are rolled. What is the probability of obtaining a sum of at least 10?
 - ◇ Let E : "obtaining at least 10" $\Leftrightarrow E = E_1 \cup E_2 \cup E_3$, where:
 - ◇ E_1 : "obtaining 10" = $\{(4, 6), (5, 5), (6, 4)\} \Rightarrow |E_1| = 3$
 - ◇ E_2 : "obtaining 11" = $\{(5, 6), (6, 5)\} \Rightarrow |E_2| = 2$
 - ◇ E_3 : "obtaining 12" = $\{(6, 6)\} \Rightarrow |E_3| = 1$
 - ◇ These events are mutually exclusive:
 $E_1 \cap E_2 = E_1 \cap E_3 = E_2 \cap E_3 = \emptyset$.

Complete System of Events: Example (continued)

- ◇ The sample space is:

$$\begin{aligned}\Omega &= \{(i, j) \mid 1 \leq i \leq 6, 1 \leq j \leq 6\} \\ &= \{(1, 1), (1, 2), \dots, (6, 6)\}\end{aligned}$$

so the number of possible outcomes is $|\Omega| = 36$.

- Recall: $\Pr(A) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$

- ◇ Therefore,

$$\begin{aligned}\Pr(E) &= \Pr(E_1 \cup E_2 \cup E_3) \\ &= \Pr(E_1) + \Pr(E_2) + \Pr(E_3) = \frac{3}{36} + \frac{2}{36} + \frac{1}{36} = \frac{1}{6}.\end{aligned}$$

Conditional Probabilities: Introduction

- Let A and B be two non-mutually exclusive events ($A \cap B \neq \emptyset$).
 - ◇ **Example (Medical context):** A = "patient has appendicitis", B = "patient presents right lower quadrant abdominal pain (RLQ pain): signe clinique (douleur de la fosse iliaque droite)".
- The probability of A given that B has occurred is written as $\Pr(A|B)$.
 - ◇ Example: the probability that a patient has appendicitis given that he or she presents RLQ pain.
 - ◇ In general, during diagnosis, the probability that a patient has a particular disease changes as symptoms and test results become known.

Conditional Probabilities: Population Perspective

- Consider a population P .
- Let A be an event with probability $\Pr(A) > 0$.
- What happens to $\Pr(A)$ when we restrict ourselves to a subpopulation defined by B ?
 - Example 1: A = "height between 170-175 cm"; subpopulation = "males".
 - Example 2 (medical): A = "presence of appendicitis"; subpopulation = "patients with RLQ pain".
- The event B defines the subpopulation-it's the **conditioning event**.
 - Example: B = "being male"; or B = "presenting symptom S ".
- The complete set of possible outcomes can be described as a product space:

$$\{(M, S), (M, \bar{S}), (\bar{M}, S), (\bar{M}, \bar{S})\},$$

where M = "disease present" and S = "symptom present".

Conditional Probability: Notation

- $\Pr(A|B)$ denotes:
 - the probability of A among individuals for whom B is true;
 - equivalently, "the probability of A given that B occurred".
- **Common mistake:** confusing $\Pr(A|B)$ with $\Pr(A \cap B)$.
- These two are related but not equal:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \quad \text{where } \Pr(B) > 0.$$

Conditional Probability: Formula and Properties

- The formal definition is:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\text{Number of outcomes with both } A \text{ and } B}{\text{Number of outcomes with } B}$$

- $\Pr(B) \neq 0$ is required.
- $\Pr(A|B)$ is a valid probability measure over A :
 - Since $A \cap B \subset B$, we have $\Pr(A|B) \leq 1$.
 - If A_1 and A_2 are disjoint (at least among B):

$$\Pr((A_1 \cup A_2)|B) = \Pr(A_1|B) + \Pr(A_2|B).$$

Example in Medicine

- Let:

A : "Patient has appendicitis",

B : "Patient has right lower quadrant pain (RLQ pain)".

- Suppose in a hospital study:

$$\Pr(A) = 0.05, \quad \Pr(B) = 0.20, \quad \Pr(A \cap B) = 0.04.$$

Then,

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.04}{0.20} = 0.20.$$

- Interpretation: among patients presenting RLQ pain, 20% have appendicitis.

Conditional Probabilities: Example 1 (Finite Ω)

- Two fair dice are rolled. Given that the **sum is 6**, what is the probability that **one of the dice shows a 2**?

- There are 36 equally likely outcomes in total.
- Let B : "the sum of the two dice equals 6".
 $B = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$.
- Let A : "at least one of the two dice shows 2".
 $A = \{(2, 1), (1, 2), (2, 2), (2, 3), (3, 2), (2, 4), (4, 2), (2, 5), (5, 2), (2, 6), (6, 2)\}$.
- Then $A \cap B = \{(2, 4), (4, 2)\}$, so $|A \cap B| = 2$.
- Thus,

$$\Pr(A \cap B) = \frac{2}{36} = 0.056, \quad \Pr(B) = \frac{5}{36}, \quad \Pr(A) = \frac{11}{36}.$$

- Therefore,

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{2/36}{5/36} = \frac{2}{5} = 0.4.$$

Conditional Probabilities: Example 2 (Drawing Balls)

- Two balls are drawn successively **without replacement** from a bag containing **10** white balls and **6** black balls. What is the probability that **both balls are white**?
- Define:
 - ◇ F : "the first ball is white" $\Rightarrow \Pr(F) = \frac{10}{16}$.
 - ◇ G : "the second ball is white" $\Rightarrow \Pr(G|F) = \frac{9}{15}$.
 - ◇ E : "both balls are white" $= F \cap G$.
- Then, by the multiplication rule:

$$\Pr(E) = \Pr(G|F) \times \Pr(F) = \frac{9}{15} \times \frac{10}{16} = \frac{3}{8} = 0.375.$$

Conditional Probabilities: Example 3 (Medical Diagnosis)

Example: Diagnostic Scenario

Among patients visiting a doctor, the probability of having a disease M is $\Pr(M) = 0.2$. A symptom S always appears when the disease is present (perfect indicator), but not all patients with M show it. It is known that 10% of all patients have both the disease and the symptom. What is the probability that a patient with M shows symptom S ?

- Let M : "having the disease", S : "having the symptom".
- Given:

$$\Pr(M) = 0.2, \quad \Pr(M|S) = 1, \quad \Pr(M \cap S) = 0.1.$$

Conditional Probabilities: Example 3 (Medical Diagnosis)

- By definition of conditional probability:

$$\Pr(S|M) = \frac{\Pr(M \cap S)}{\Pr(M)} = \frac{0.1}{0.2} = 0.5.$$

- Interpretation: Among patients who actually have the disease, 50% show the symptom S .

Multiplication Rule of Conditional Probability

- From the definition:

$$\Pr(B|A) = \frac{\Pr(A \cap B)}{\Pr(A)} \Rightarrow \Pr(A \cap B) = \Pr(B|A) \times \Pr(A).$$

- Similarly,

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} \Rightarrow \Pr(A \cap B) = \Pr(A|B) \times \Pr(B).$$

- Therefore, the probability that both A and B occur is:

Multiplication Rule

$$\Pr(A \cap B) = \Pr(B|A) \times \Pr(A) = \Pr(A|B) \times \Pr(B).$$

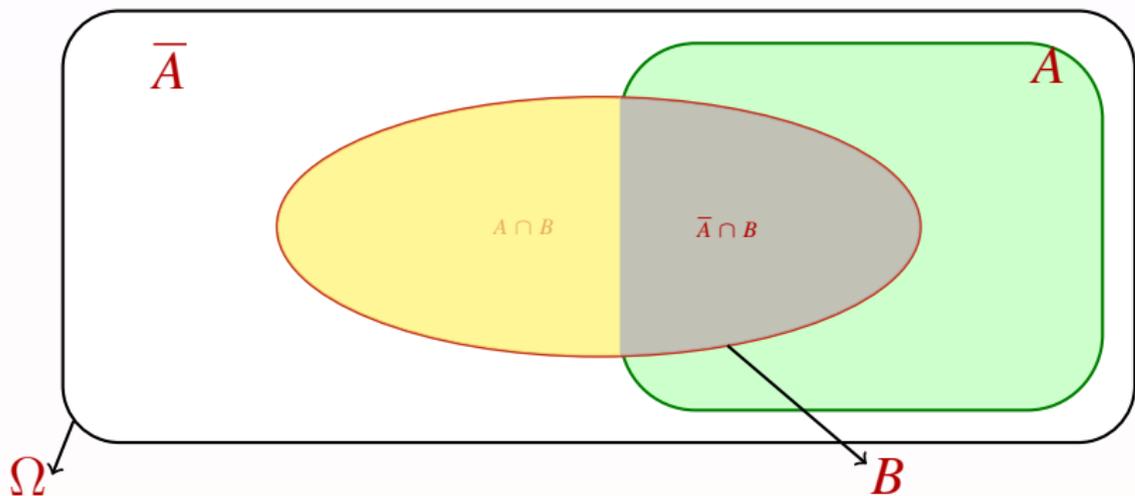
- In general, for three non-exclusive events A, B, C :

$$\Pr(A \cap B \cap C) = \Pr(A) \times \Pr(B|A) \times \Pr(C|A \cap B).$$

Law of Total Probability: Special Case

- Let $B \subset \Omega$. When the sample space can be written as $\Omega = A \cup \bar{A}$, we have:
 - ◊ $B = B \cap \Omega = B \cap (A \cup \bar{A}) = (B \cap A) \cup (B \cap \bar{A})$
 - ◊ The two events are mutually exclusive since $(B \cap A) \cap (B \cap \bar{A}) = \emptyset$ because $A \cap \bar{A} = \emptyset$.
- Therefore:

$$\Pr(B) = \Pr(A \cap B) + \Pr(\bar{A} \cap B) = \Pr(B|A)\Pr(A) + \Pr(B|\bar{A})\Pr(\bar{A})$$



Law of Total Probability: Example 1

- A population consists of **40% men** and **60% women**. Among them, **50% of men** and **30% of women** are smokers. What is the probability that a randomly selected person is a smoker?

- ◇ S : "the person is a smoker"
- ◇ H : "the person is a man"
- ◇ F : "the person is a woman"
- ◇ Given:

$$\Pr(H) = 0.4, \quad \Pr(F) = 0.6, \quad \Pr(S|H) = 0.5, \quad \Pr(S|F) = 0.3$$

- Since $\Omega = H \cup F$ with $H \cap F = \emptyset$, the total probability of being a smoker is:

$$\begin{aligned}\Pr(S) &= \Pr(S \cap H) + \Pr(S \cap F) \\ &= \Pr(S|H)\Pr(H) + \Pr(S|F)\Pr(F) \\ &= (0.5)(0.4) + (0.3)(0.6) = 0.38.\end{aligned}$$

- **Interpretation:** There is a **38%** chance that a randomly chosen person from the population is a smoker.

Law of Total Probability: General Case

- Let (A_1, A_2, \dots, A_n) be a **complete system of mutually exclusive events** such that:

$$A_i \cap A_j = \emptyset \quad (i \neq j) \quad \text{and} \quad \bigcup_{i=1}^n A_i = \Omega.$$

- For any event B :

$$B = B \cap \Omega = B \cap (A_1 \cup A_2 \cup \dots \cup A_n) = (B \cap A_1) \cup (B \cap A_2) \cup \dots \cup (B \cap A_n).$$

- Since the events (A_i) are mutually exclusive, the intersections $(B \cap A_i)$ are also mutually exclusive. Hence,

$$\Pr(B) = \Pr(B \cap A_1) + \Pr(B \cap A_2) + \dots + \Pr(B \cap A_n).$$

- By the **Multiplication Rule**:

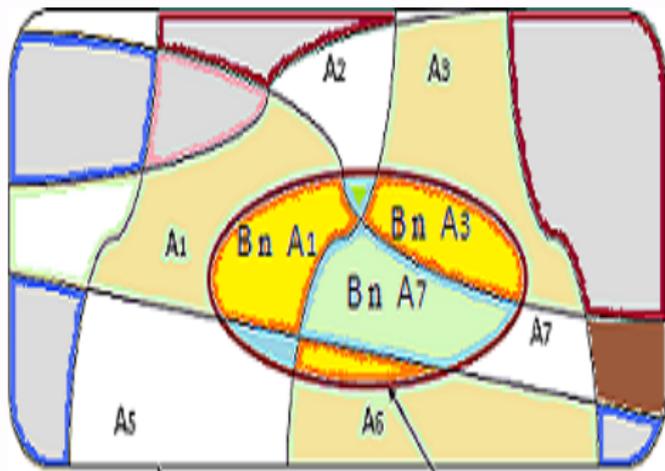
$$\Pr(B \cap A_i) = \Pr(B|A_i) \times \Pr(A_i).$$

Law of Total Probability: General Case

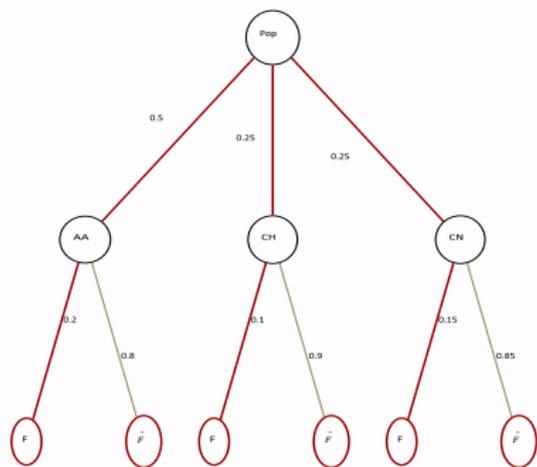
- Therefore, the **Law of Total Probability** can be written as:

Law of Total Probability

$$\Pr(B) = \sum_{i=1}^n \Pr(B \cap A_i) = \sum_{i=1}^n \Pr(B|A_i) \times \Pr(A_i).$$



Law of Total Probability: Example 3 with a Tree Diagram



- A patient presents with acute abdominal pain.
- Three possible pathologies:
 - **AA**: Acute appendicitis (50% of cases) $\Rightarrow \Pr(AA) = 0.5$
 - **CH**: Cholecystitis (25% of cases) $\Rightarrow \Pr(CH) = 0.25$
 - **CN**: Colonic necrosis (25% of cases) $\Rightarrow \Pr(CN) = 0.25$
- Among these patients:
 - 20% of AA cases have fever $\Rightarrow \Pr(F|AA) = 0.2$
 - 10% of CH cases and 15% of CN cases have fever $\Rightarrow \Pr(F|CH) = 0.1$, $\Pr(F|CN) = 0.15$

Law of Total Probability: Example 3 with a Tree Diagram

- What is the probability of fever given acute abdominal pain?

$$\begin{aligned}\Pr(F) &= \Pr(F|AA)\Pr(AA) + \Pr(F|CH)\Pr(CH) + \Pr(F|CN)\Pr(CN) \\ &= 0.2 \times 0.5 + 0.1 \times 0.25 + 0.15 \times 0.25 = 0.1625\end{aligned}$$

- Therefore, the probability of fever is approximately **16.2%**.

Rule:

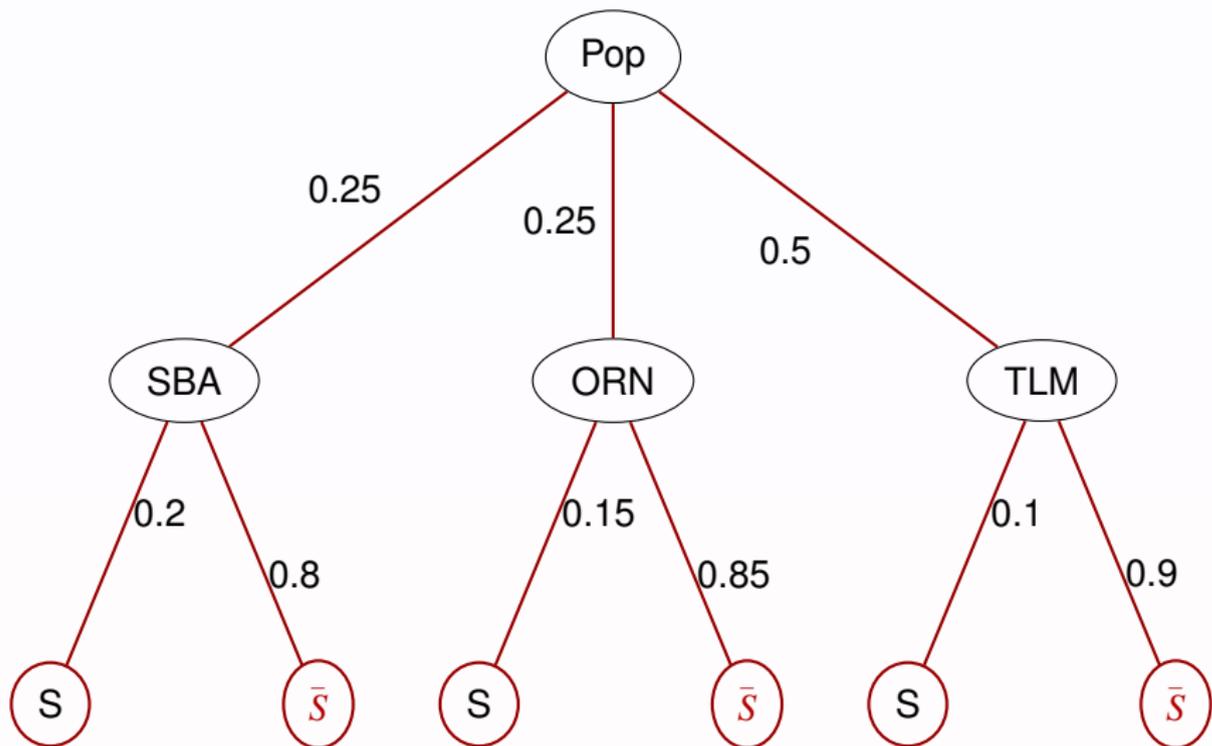
- According to the multiplication rule, the probability that a particular path of the tree occurs is the **product of the probabilities** of each branch along that path.
- Since the different paths are **mutually exclusive**, the total probability of an outcome (for example, being accepted) is the **sum of the probabilities** of all paths leading to that outcome state F (accepted).

Law of Total Probability: Example 4 with a Tree Diagram

- The success rates in the entrance exam for the three university hospitals (CHUs) of **Sidi Bel Abbès (SBA)**, **Oran**, and **Tlemcen** are respectively:
 $\Pr(\text{Success}|\text{SBA}) = 0.2$, $\Pr(\text{Success}|\text{ORN}) = 0.15$,
 $\Pr(\text{Success}|\text{TLM}) = 0.1$.
- It is also known that among all students in the western region:

$$\Pr(\text{SBA}) = \frac{1}{4}, \quad \Pr(\text{Oran}) = \frac{1}{4}, \quad \Pr(\text{Tlemcen}) = \frac{1}{2}.$$

- What is the probability that a student from the western region passes the exam?



Law of Total Probability: Example 4 with a Tree Diagram

- Let S denote the event "Success" and F the event "Failure". Then, according to the law of total probability:

$$\begin{aligned}\Pr(R) &= \Pr(S \cap SBA) + \Pr(S \cap ORN) + \Pr(S \cap TLM) \\ &= \Pr(S|SBA)\Pr(SBA) + \Pr(S|ORN)\Pr(ORN) + \Pr(S|TLM)\Pr(TLM) \\ &= 0.2 \times \frac{1}{4} + 0.15 \times \frac{1}{4} + 0.1 \times \frac{1}{2} = 0.1375.\end{aligned}$$

- Therefore, the probability that a student from the western region passes the exam is approximately **13.75%**.

- **Multiplication rule:**

$$\Pr(A \cap B) = \Pr(A|B) \times \Pr(B) = \Pr(B|A) \times \Pr(A)$$

- Using the law of total probability:

$$\begin{aligned}\Pr(A) &= \Pr(A \cap B) + \Pr(A \cap \bar{B}) \\ &= \Pr(A|B)\Pr(B) + \Pr(A|\bar{B})\Pr(\bar{B})\end{aligned}$$

- We obtain **Bayes' formula:**

$$\Pr(B|A) = \frac{\Pr(A|B) \times \Pr(B)}{\Pr(A)} = \frac{\Pr(A|B)\Pr(B)}{\Pr(A|B)\Pr(B) + \Pr(A|\bar{B})\Pr(\bar{B})}.$$

valid whenever $\Pr(A) \neq 0$.

Bayes' Theorem: Interpretation

- This formula is also called the "**theorem of the probability of causes**", as it allows one to **reverse the conditioning**.
- **Interpretation:**
 - ◇ A represents an **effect or consequence**, and B a **possible cause**.
 - ◇ Bayes' theorem allows us to infer the most likely **cause** from an observed **effect**.
- To compute a conditional probability, we can use:
 - ◇ The **definition** of conditional probability; or
 - ◇ The **Bayes' theorem**.

Exercise

A disease M occurs in two forms M_1 and M_2 , with respective probabilities

$$\Pr(M_1) = 0.2, \quad \Pr(M_2) = 0.8.$$

A symptom S appears in 80% of cases of M_1 and in 10% of cases of M_2 . What is the probability that a patient **who presents symptom S actually suffers from form M_1** ?

Bayes' Theorem: Exercise Solution

- M_1 : "Patient has disease of type M_1 "
- M_2 : "Patient has disease of type M_2 "
- S : "Patient shows symptom S "
- $\Pr(S|M_1) = 0.8$ and $\Pr(S|M_2) = 0.1$
- Since $\overline{M_1} = M_2$, we have:

$$\begin{aligned}\Pr(M_1|S) &= \frac{\Pr(S|M_1)\Pr(M_1)}{\Pr(S|M_1)\Pr(M_1) + \Pr(S|\overline{M_1})\Pr(\overline{M_1})} \\ &= \frac{0.8 \times 0.2}{0.8 \times 0.2 + 0.1 \times 0.8} = \frac{2}{3}.\end{aligned}$$

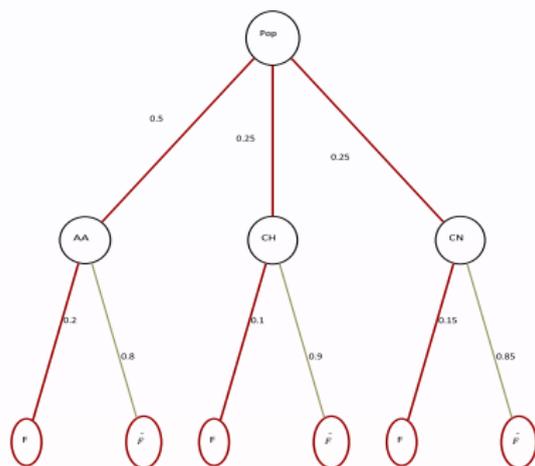
- Thus, there is a **66.7% probability** that a patient with symptom S has form M_1 .

Bayes' Theorem: General Case

- Let $(A_i)_{i=1,\dots,n}$ be a **complete system of events** (a partition of the sample space E), and let B be another event.
- For each A_i , we have $\Pr(A_i|B) = \frac{\Pr(B \cap A_i)}{\Pr(B)}$.
- Using the law of total probability, we obtain the **general Bayes' formula**:

$$\Pr(A_i|B) = \frac{\Pr(A_i)\Pr(B|A_i)}{\Pr(A_1)\Pr(B|A_1) + \Pr(A_2)\Pr(B|A_2) + \dots + \Pr(A_n)\Pr(B|A_n)}.$$

Bayes' Theorem: General Example



- A patient presents with **fever** and **acute abdominal pain**.
- What is the most probable cause among the possible pathologies?
- Recall: $\Pr(F) = 0.162$.

Bayes' Theorem: Example (continued)

- The probability that a patient with fever suffers from pathology **AA** is:

$$\begin{aligned}\Pr(AA|F) &= \frac{\Pr(F|AA)\Pr(AA)}{\Pr(F)} \\ &= \frac{\Pr(F|AA)\Pr(AA)}{\Pr(F|AA)\Pr(AA) + \Pr(F|CH)\Pr(CH) + \Pr(F|CN)\Pr(CN)} \\ &= \frac{0.2 \times 0.5}{0.162} = 0.62.\end{aligned}$$

- Hence, $\Pr(CH|F) = 0.15$ and $\Pr(CN|F) = 0.23$.

Independent Events: Introduction

- Let A and B be two events with known probabilities:
 $\Pr(A) > 0$ and $\Pr(B) > 0$.
- We want to determine the probability that A and B occur simultaneously: $\Pr(A \cap B) = ?$
- This probability could be estimated experimentally from relative frequencies, but that process is often long and costly.
- In some cases, however, this probability can be deduced directly from $\Pr(A)$ and $\Pr(B)$, when the two events are independent.



Independent Events: Definition 1

- Event A is said to be **independent** of event B if the occurrence (or non-occurrence) of B **has no influence** on the occurrence of A . That is, knowing whether B occurs does not change the probability of A .

Definition

- Events A and B are independent if $\Pr(A|B) = \Pr(A)$.
- If A and B are **not independent**, we say they are **dependent** (or related).

Independent Events: Definition 2

- From the definition of conditional probability:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)},$$

we immediately obtain the following equivalence:

Equivalent Formulation

Events A and B are independent if and only if

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B).$$

- Other equivalent forms:

$$\Pr(A|\bar{B}) = \Pr(A) \quad \text{and} \quad \Pr(B|A) = \Pr(B).$$

Independence vs. Incompatibility

- One must **not confuse** independent events with mutually exclusive (incompatible) events.
 - ◇ If $A \cap B = \emptyset$ (mutually exclusive):
 - If B occurs, A cannot occur (and vice versa). Thus, the occurrence of B directly affects A , it prevents it. Indeed:
 $\Pr(A \cap B) = 0 \Rightarrow \Pr(A|B) = 0 = \Pr(B|A)$.
 - Moreover, $\Pr(A \cap B) = 0 \neq \Pr(A)\Pr(B)$, so A and B are **not independent**.
 - ◇ If A and B are independent:
 - The occurrence of B has no influence on A (and vice versa).
 - Since $\Pr(A \cap B) = \Pr(A)\Pr(B) \neq 0$, it follows that $(A \cap B) \neq \emptyset$, thus, A and B are **compatible**.

Exercise 1

Based on health data, the following information was obtained:
Renal colic and asthma are two independent diseases.

- Prevalence (proportion of individuals affected at a given time) of asthma: **0.0098**
- Prevalence of renal colic: **0.002**
- Prevalence of pulmonary infections (PI): **0.10**
- Frequency of PI among asthmatic patients: **0.3**
- **40%** of asthmatics are smokers; among these, **30%** present cardiovascular diseases, while **10%** of non-smoking asthmatics have cardiovascular diseases.

Questions

- 1 What is the probability of being asthmatic *or* having a pulmonary infection?
- 2 What is the probability of having either asthma or renal colic?
- 3 If a patient has a pulmonary infection, what is the probability that they are asthmatic? (i.e., the frequency of asthmatics among those with PI)
- 4 What is the frequency of cardiovascular diseases among asthmatics?
- 5 What is the frequency of non-smokers among asthmatics who have no cardiovascular disease?

Exercise 2

A disease A can occur in three forms, denoted A_1 , A_2 , and A_3 . The respective probabilities of each form are $\frac{1}{2}$, $\frac{1}{6}$, and $\frac{1}{3}$. To refine the diagnosis for a patient with disease A , a test E is performed. The test result is positive in:

- 10% of cases with form A_1 ,
 - 20% of cases with form A_2 , and
 - 90% of cases with form A_3 .
- 1 What is the probability that the test E is negative for a randomly selected patient with disease A ?
 - 2 If a patient with disease A tests positive, what is the probability that they have form A_1 ? A_2 ? A_3 ?

Exercise 3

In a pharmaceutical company, a tablet is considered *perfect* if its mass (in grams) lies in the interval $[1.2, 1.3]$. The probability that a tablet is compliant is 0.98 . Let:

- A : "the tablet is compliant",
- B : "the tablet is rejected".

The control mechanism is as follows:

- A compliant tablet is accepted with probability 0.98 .
- A non-compliant tablet is rejected with probability 0.99 .

Questions

- 1 Compute $\Pr(B/A)$, then $\Pr(A \cap B)$.
- 2 Compute the probability that a tablet is rejected.
- 3 Compute the probability that a tablet is compliant, given that it was rejected.

Exercise 4

It is estimated that a person has a 6 out of 10 chance of having a certain disease. Two screening tests are performed:

- The first test is positive in 70% of diseased individuals and 20% of healthy individuals.
- The second test is positive in 90% of diseased individuals and 30% of healthy individuals.

The two tests are assumed to be independent, both among the diseased and among the healthy (conditional independence).

Questions

- 1 A randomly chosen person tests positive on the first test. What is the probability that they are diseased?
- 2 A randomly chosen person tests positive on both tests. What is the probability that they are diseased?
- 3 What is the probability that the second test is positive, given that the first one was positive?

Exercise 5

All numerical results should be given to two decimal places.

A clinical examination shows that an individual has a pathology P , which can appear in three forms: A , B , and C . To determine which form is present, specific diagnostic tests are performed. If the test is positive, the corresponding form is certainly present; if negative, nothing can be concluded.

When the pathology P is present:

- The proportions of forms A , B , and C in the population are 50%, 30%, and 20%, respectively.
- The test corresponding to each form is positive in 70% of individuals with form A , 90% of those with form B , and 10% of those with form C .

We consider only individuals who have pathology P .

Questions (1)

For a randomly chosen individual:

1. The probability of having pathology P is:
 $(A) = 0.5, (B) = 0.64, (C) = 0.7, (D) = 0.9, (E) = 1$
2. The probability of having form A :
 $(A) = 0.2, (B) = 0.3, (C) = 0.5, (D) = 0.7, (E) = 0.9$
3. If the individual has form A , the probability that the corresponding test is negative: $(A) = 1, (B) = 0.3, (C) = 0.7, (D) = 0, (E)$ indeterminate.
4. If the test for form A is performed, the probability it is positive: $(A) = 0, (B) = 0.35, (C) = 0.64, (D) = 1, (E)$ indeterminate.

Questions (2)

5. If the test corresponding to form A is positive, the probability that the individual does **not** have form A is:
 $(A) = 0, (B) = 0.2, (C) = 0.3, (D) = 0.5, (E) = 1$
6. Since form A is the most common, its test is performed first. If it is negative, the probability that the individual has form A is:
 $(A) = 0.15, (B) = 0.23, (C) = 0.35, (D) = 0.65, (E) = 1$
7. If the test for A is negative, we perform tests for B and C , which are also negative. Then, the probability that the individual has form A is:
 $(A) = 0.10, (B) = 0.20, (C) = 0.23, (D) = 0.27, (E) = 0.90$

Exercice 5

On donnera une valeur approchée de tous les résultats à 10^{-2} près. L'examen clinique d'un individu montre qu'il est atteint d'une pathologie P pouvant présenter 3 formes A, B, C qu'on cherche à déterminer. On réalise pour cela des tests supplémentaires spécifiques à chacune de ces formes. Si le test est positif, la forme correspondante de la pathologie est certainement présente, sinon, on ne peut rien affirmer. On sait que, lorsque cette pathologie est présente, les trois formes A, B, C apparaissent chez les individus dans des proportions 50%, 30% et 20%. On sait aussi que 70% des sujets atteints de la forme A ont un test correspondant positif, que 90% atteints de B ont un test correspondant positif et que 10% atteints de C ont un test correspondant positif. On considère que la population est constituée d'individus atteints de la pathologie P.

On choisit un individu au hasard. Alors:

1. La probabilité d'avoir la pathologie P est:

$$(A) = 0.5, \quad (B) = 0.64, \quad (C) = 0.7, \quad (D) = 0.9, \quad (E) = 1.$$

2. La probabilité d'avoir la forme A est:

$$(A) = 0.2, \quad (B) = 0.3, \quad (C) = 0.5, \quad (D) = 0.7, \quad (E) = 0.9.$$

3. Si l'individu a la forme A, alors probabilité que le test correspondant soit négatif est:

$$(A) = 1, \quad (B) = 0.3, \quad (C) = 0.7, \quad (D) = 0, \quad (E) \text{ on ne peut rien dire.}$$

4. Si on réalise un test correspondant à la forme A sur l'individu, alors la probabilité qu'il soit positif est:

$$(A) = 0, \quad (B) = 0.35, \quad (C) = 0.64, \quad (D) = 1, \quad (E) \text{ on ne peut rien dire.}$$

5. Si le test correspondant à la pathologie du forme A est positif, alors probabilité que l'individu n'a pas la forme A est:
 $(A) = 0, (B) = 0.2, (C) = 0.3, (D) = 0.5, (E) = 1.$
6. La forme A étant plus présente, on commence par réaliser le test correspondant. Le test est négatif. Alors, la probabilité que cet individu présente la forme A est:
 $(A) = 0.15, (B) = 0.23, (C) = 0.35, (D) = 0.65, (E) = 1$
7. Le test pour A n'ayant rien donné, on réalise les deux autres tests correspondant aux formes B et C. Là encore, les tests sont négatifs. Alors la probabilité que cet individu présente la forme A est:
 $(A) = 0.1, (B) = 0.2, (C) = 0.23, (D) = 0.27, (E) = 0.9.$