Probability

Concepts of Probability and Combinatorial Analysis

Benchikh Tawfik

Faculty of Medicine

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Probability and Statistics

Probability

- Concerns **populations**, **models**, and **theoretical concepts**.
- Deals with quantities that **cannot be directly measured**.

Statistics

- Concerns **samples**, the **real world**, and **practical applications**.
- Involves quantities that **can be measured** on individuals.

Probability builds models; Statistics uses data to test them.

Probability

- Probability theory deals with **random phenomena** (also called **random experiments**).
- Chapters 2 and 3 introduce the **fundamental concepts of probability theory**.
- These chapters focus on the concepts essential for the **statistical applications** developed in later sections.

Objectives: Probability

Specific Objectives:

- Learn how to use **frequency distribution tables** to derive simple probabilities.
- Compute **probabilities of compound events**.
- Calculate **conditional probabilities** by applying the rules of probability.

From data ? to models ? to inference.

Course Outline

1. Notations and Definitions
2. Probability on a Finite or Infinite Space
   - Definition of a Probability
   - Probabilities on a Finite Sample Space
   - Probabilities on a Countably Infinite Sample Space
   - Probabilities on an Uncountably Infinite Set ($\mathbb{R}$)
3. Combinatorial Analysis
   - Fundamental Principle
   - Arrangements
   - Arrangements with Repetition
   - Combinations
4. Exercices

Random Phenomena-I

- **Deterministic phenomena:** These are predictable and produce the same outcome each time the experiment is repeated. Their behavior does not depend on any probability law.
- **Random phenomena:** These are phenomena for which the outcome cannot be predicted in advance. However, when repeated many times, they display a certain regularity and follow probability laws.

Random Phenomena-II

- $\surd$ A typical example is the **toss of a coin**.
- $\surd$ We cannot predict the result of any single toss, but after many repetitions, we will obtain approximately 50% heads (assuming the coin is fair).

Random Phenomena-III

Consider n successive tosses of the same coin:

Number of tosses	10	100	200	500	1000	10000
Number of heads	3	45	95	260	491	4995
Relative frequency	0.30	0.45	0.475	0.52	0.491	0.4995

- Hence, the relative frequencies tend to 0.5 (i.e., 50%), which corresponds to the probability of obtaining heads, as the number of tosses tends to infinity.

Random Phenomena-IV

- Random phenomena exhibit a certain **regularity**. This regularity allows us to predict the **frequency of occurrence** of an event.
- **Modeling** such phenomena (i.e., finding their underlying **probability laws**) is the main objective of **probability theory**.
- A random experiment is called a **trial**.
- We restrict our attention to experiments that can be **repeated under identical conditions**.

Random Experiment and Sample Space

- **Random Experiment:**
  - √ Examples: rolling a die, measuring blood glucose in 100 individuals.
  - √ Experiment $\rightarrow$ observations $\rightarrow$ statistical analysis.
  - √ Study of possible outcomes $\rightarrow$ probability theory.
- **Sample Space (set of all possible outcomes):**
  - √ Denoted by $\Omega = \{\omega_1, \ldots, \omega_n\}$, it represents the set of all possible results of the experiment.
  - √ Ω may be finite or infinite, countable or uncountable.

Events 1

- An event is a subset of possible outcomes, usually denoted by capital Latin letters such as A, B, etc.
  - ◇ Example: if $E = \{1,2,3,4,5,6\}$, then the event "even result" is $\{2,4,6\}$. There are $2^6 = 64$ possible events.
  - ◇ An event occurs if the result of the experiment belongs to the corresponding subset.
    - √ Example: the event "even result" occurs when we obtain 2, 4, or 6.

Events 2

- Special cases:
  - $\{\omega_i\}$: realization or elementary event.
  - Compound event: obtained by combining several elementary events (simultaneous or successive occurrence of two or more simple events).
  - The set of all possible events is the **event space**, denoted $\mathcal{P}(\Omega)$-the power set of Ω.
  - $\emptyset$: the empty set-the impossible event.
  - Ω: the certain event.

Event-Example 1

1. Tossing a coin is a random experiment. The sample space is $\Omega = \{H = \text{Heads}, T = \text{Tails}\}$.
   - "Getting Tails" is an elementary event: $E_1 = \omega_1 = \{T\}$.
   - "Getting Heads" is an elementary event: $E_2 = \omega_2 = \{H\}$.

Event-Example 2

Example

2. Tossing a coin twice in succession: $\Omega = \{HH, HT, TH, TT\}$.
    - "Getting two tails" is an elementary event: $E_1 = \omega_1 = \{TT\}$.
    - "Getting at least one head" is a compound event: $\{HH, HT, TH\}$.

Event-Example 3

Examples

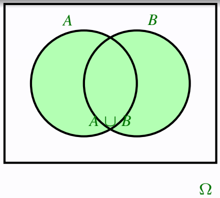
3. Rolling a die twice: $\Omega = \{(i, j) \mid 1 \leq i \leq 6,\ 1 \leq j \leq 6\}$. The event "the sum equals 6" is a subset of Ω.
4. The birth weight of newborns (in grams) can be represented as $\Omega = [1000, 6000]$. Then $[1500, 2500] \subset [1000, 6000]$ defines an event.
5. The lifetime of a light bulb: $\Omega = \mathbb{R}_+$.

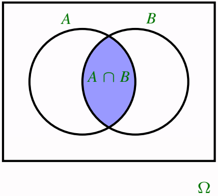
Event-Example 4

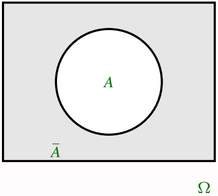
6. Birth: after a birth, the possible outcomes are "girl" or "boy". The sex of the child is the **random phenomenon**, since the result (girl or boy) cannot be predicted with certainty.

Operations on Events

- **Union:** $A \cup B$ Occurs if either A or B (or both) occur.
- **Intersection:** $A \cap B$ Occurs only if both A and B occur.
- **Complement:** $\bar{A}$ (or A^c) Occurs when A does not occur.
- **Disjoint (mutually exclusive) events:** $A \cap B = \emptyset$







Rules for Combining Events (1)

- If A and B are two events, then:
  - $A \cap B$ is an event (the intersection)
  - $A \cup B$ is an event (the union)
  - $C_\Omega A = \overline{A}$ is an event (the complement)
- If Ω is **finite** or **countably infinite**, every subset of E is an event.
- If Ω is **uncountably infinite** (for example, $\mathbb{R}$), an event is an interval or a combination of intervals.

Rules for Combining Events (2)

a) The event "A **or** B" = "$A \cup B$": occurs if at least one of the events A or B occurs.

- ◇ Example: "having a heart attack" **or** "being diabetic."

b) The event "A **and** B" = "$A \cap B$": occurs only if both A and B occur.

- ◇ Example: "having a heart attack" **and** "being diabetic."

c) The event **not** A, denoted $\overline{A}$, occurs if and only if A does not occur.

Rules for Combining Events (3)

d) Compatible events: events that can occur simultaneously.

e) Events A and B are incompatible or mutually exclusive $\Leftrightarrow A$ and B are disjoint $\Leftrightarrow A \cap B = \emptyset$.

- If A occurs, B cannot occur, and vice versa-meaning that A and B cannot happen together.

f) A implies $B \Leftrightarrow A \subset B$: if A occurs, then B necessarily occurs.

g) The event "A without B" $\Leftrightarrow A - B$: A occurs without B occurring.

Example: Combining Events

- A die is rolled.
  - The **sample space** is $\Omega = \{1, 2, 3, 4, 5, 6\}$.
  - Event A: **"getting an even number** $\Rightarrow A = \{2, 4, 6\} \subset \Omega$.
  - Event B: **"getting a prime number"** $\Rightarrow B = \{2, 3, 5\}$.
  - Event C: **"getting a 3"** $\Rightarrow C = \{3\}$.

Example: Combining Events (continued)

- $A \cup B = \{2, 3, 4, 5, 6\}$: **"getting an even or prime number."**
- $A \cap B = \{2\}$: **"getting a number that is both even and prime."**
- $\overline{C} = \{1, 2, 4, 5, 6\}$: **"getting a number other than 3."**
- $A \cap C = \emptyset$: A and C are **mutually exclusive**.

Rules for Combining Events: Example

- In the previous example, Ω was finite and therefore countable.
- Ω can also be infinite and countable, as in the following case:

  1. A coin is tossed repeatedly until the first head appears.
     - $\Omega = \{1, 2, \ldots, n, \ldots\}$, since a head can appear on the 1st, 2nd, nth, etc. toss.

- Ω may also be infinite and uncountable, for example:

  2. A dart is thrown at a large target. If the dart is considered to be infinitely thin,
     - $\Omega =$ {the surface of the target}, which is infinite and uncountable.

Definition of Probability

- Probability theory does not allow us to calculate all probabilities directly.
- However, it enables us to compute the probabilities of combinations of events when the probabilities of simpler events are known.
- The main concept used here: **probability = limit of relative frequency**.

Probability in a Finite Sample Space: Definition

Definition

To each event A, we associate a number denoted by $\mathbf{P}(A)$, called the **probability of** A. This number represents the degree of likelihood we assign *a priori* to A, before the experiment is carried out.

Mathematically, we define:

$$\begin{aligned} \mathbf{P} : (\Omega, \mathcal{A}) &\rightarrow [0, 1] \\ A &\mapsto \mathbf{P}(A) \end{aligned}$$

Basic Definitions

- Any event A such that $\mathbf{P}(A) = 1$ is said to be **almost certain**.
- Any event B such that $\mathbf{P}(B) = 0$ is said to be **almost impossible**.

Probability as a Limit of Frequency

- In practice, probability can be viewed as the **long-run frequency** of occurrence of an event when the number of trials becomes very large. That is, probability is the limit of relative frequency.
  - ◇ Example: tossing a fair coin n times

Number of tosses	10	100	200	500	1000	10000
Number of heads	3	45	95	260	491	4995
Relative frequency	0.30	0.45	0.475	0.52	0.491	0.4995

  - ◇ Let $f_n(A) = \dfrac{\text{number of occurrences of } A}{n}$ be the relative frequency of event A. Then:

$$\mathrm{P}(A) = \lim_{n \to \infty} f_n(A)$$

Probability: Remark

- √ This approach is not always possible, for instance, when considering events that have never been observed (e.g., rare astronomical phenomena).
- √ To study the properties of these probability values, we rely on the properties of observed frequencies.

Axioms of Probability

- From the properties of frequencies, we derive the following axioms:

(P0) $0 \le \mathrm{P}(A) \le 1$;

(P1) $\mathrm{P}(\Omega) = 1$;

(P2) If $A \cap B = \emptyset$, then $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$.

- It follows that:
  - $\mathrm{P}(\emptyset) = 0$.
  - $\mathrm{P}(A) + \mathrm{P}(\overline{A}) = 1$.
  - $\mathrm{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mathrm{P}(A_i)$, if the A_i are pairwise disjoint.
  - $\mathrm{P}(A \cup B) + \mathrm{P}(A \cap B) = \mathrm{P}(A) + \mathrm{P}(B)$.
  - If $A \subset B$, then $\mathrm{P}(A) \le \mathrm{P}(B)$.

Probabilities on a Finite Sample Space

Let $\Omega = \{\omega_1, \ldots, \omega_n\}$ be a finite sample space.

- We must assign a probability to each elementary event $\{\omega_i\}$:

$$\mathbf{P}(\{\omega_i\}) = \mathbf{P}(\omega_i), \quad \text{for all } \omega_i \in \Omega.$$

  - ◇ $\mathbf{P}(\omega_i) \geq 0$
  - ◇ $\sum_{i=1}^{n} \mathbf{P}(\omega_i) = 1$ (where n is the number of elementary events)

- Example: if $A = \{\omega_1, \omega_4, \omega_5\}$, then

$$\mathbf{P}(A) = \mathbf{P}(\omega_1) + \mathbf{P}(\omega_4) + \mathbf{P}(\omega_5).$$

- In general, $\mathbf{P}(A) = \sum_{\omega_j \in A} \mathbf{P}(\omega_j)$.

Special Case: Uniform Probability

Consider the experiment of rolling a die. Then $\Omega = \{1, 2, 3, 4, 5, 6\}$. If the die is fair (not biased), each of the six faces has the same probability of appearing.

- Among the six possible outcomes, only one corresponds to getting a "2". Hence, $\mathbf{P}(\text{getting 2}) = \frac{1}{6}$.
- Among the six outcomes, three are even numbers $\{2, 4, 6\}$. Therefore, $\mathbf{P}(\text{getting an even number}) = \frac{3}{6} = \frac{1}{2}$.

Uniform Probability Distribution

- **Equiprobable Sample Space:** Each elementary event has the same probability $1/n$.
  - If an event A contains k elements, its probability is

$$\mathrm{P}(A) = \frac{k}{n} = \frac{\text{number of favorable cases}}{\text{number of possible cases}}.$$

- **Remark:**
  - ✓ When we say an outcome is chosen "at random", we implicitly assume that the sample space is equiprobable.

Uniform Probability: Definition

Definition

The probability of an event is equal to the ratio of the number of favorable outcomes to the total number of possible outcomes, assuming all outcomes are equally likely (N):

$$\mathbf{P}(A) = p_i = \frac{\text{number of favorable cases}}{\text{number of possible cases}} = \frac{n_i}{N} = \frac{|A|}{|\Omega|},$$

where $|A|$ denotes the cardinality (number of elements) of A.

Uniform Probability: Example

Example: Drawing a Card

A standard deck has 52 cards.

- The probability of drawing a heart is:

$$\mathbf{P}(\text{Heart}) = \frac{13}{52} = \frac{1}{4}.$$

- The probability of drawing a face card (Jack, Queen, or King) is:

$$\mathbf{P}(\text{Face card}) = \frac{12}{52} = \frac{3}{13}.$$

Probabilities on a Countably Infinite Sample Space

Let $\Omega = \{\omega_1, \omega_2, \ldots\}$ be a countably infinite set.

- We must assign a probability to each elementary event $\{\omega_i\}$: $\mathbf{P}(\{\omega_i\}) = \mathbf{P}(\omega_i), \quad$ for all $\omega_i \in \Omega$.
  - $\mathrm{P}(\omega_i) \geq 0, \quad$ and $\quad \sum_{i=1}^{\infty} \mathrm{P}(\omega_i) = 1.$
- The probability of any (composite) event A is the sum of the probabilities of all ω_i that belong to A:

$$\mathbf{P}(A) = \sum_{\omega_j \in A} \mathbf{P}(\omega_j).$$

- Moreover, if the events A_i are pairwise disjoint, then

$$\mathbf{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbf{P}(A_i).$$

Example: Number of Tosses Until the First Head

Consider tossing a fair coin repeatedly until the first head appears. The sample space is: $\Omega = \{1, 2, 3, \ldots\}$, where each outcome n represents "the first head appears on the n-th toss".

- The probability of getting the first head on the n-th toss is:

$$\mathbf{P}(\omega_n) = \left(\frac{1}{2}\right)^n.$$

- Indeed, $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1$.
- Therefore, for example:

  $\mathbf{P}(\text{first head on the 3rd toss}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

Probabilities on an Uncountably Infinite Sample Space $\mathbb{R}$

- When Ω is an uncountable set such as $\mathbb{R}$, we cannot assign probabilities to each individual point, the probability of any single point is zero.
- Instead, we define probabilities for **intervals** or measurable subsets of $\mathbb{R}$.
- To do so, we use special functions that describe how probability is distributed:
  - The **Cumulative Distribution Function (CDF)** allows interval probabilities to be computed by subtraction:

$$\mathbf{P}(a < X \leq b) = F(b) - F(a),$$

where $F(x) = \mathbf{P}(X \leq x)$.

Example: Probability on $\mathbb{R}$

Example: Continuous Uniform Distribution

Suppose a random variable X is uniformly distributed on the interval $[0, 1]$:

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Then:

- $\mathbf{P}(X \leq 0.4) = \int_0^{0.4} 1\, dx = 0.4$,
- $\mathbf{P}(0.2 < X \leq 0.8) = \int_{0.2}^{0.8} 1\, dx = 0.6$.

Objective of Combinatorial Analysis

- Combinatorial analysis consists of a set of methods used to determine the number of all possible outcomes of a given experiment.
- Understanding these counting techniques is essential for computing probabilities, which form the foundation of statistics.

Fundamental Counting Principle

- If a situation (or event) can occur in n different ways, and it can be followed by another situation that can occur in m different ways, then the two situations can occur in $n \times m$ different ways, in the given order.

In general

If a process consists of k successive stages that can occur in $n_1, n_2, \ldots, n_k$ ways respectively, then the total number of possible outcomes is:

$$N = n_1 \times n_2 \times \cdots \times n_k.$$

Examples

1. When forming the executive board of an association, there are three candidates for president and five for treasurer. The number of possible boards is:$3 \times 5 = 15$.
2. Suppose there are 3 candidates for the position of deputy and 5 candidates for that of mayor. The two positions can be filled in: $3 \times 5 = 15$ different ways.
3. If a restaurant offers 4 starters, 3 main dishes, and 2 desserts, then the number of possible complete meals is: $4 \times 3 \times 2 = 24$.

Factorial $n!$

- The factorial of a positive integer n, denoted by $n!$, is defined as:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1.$$

  - For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
- For convenience, we define $0! = 1$.

Definition of an Arrangement

- Given a set of n (distinct) objects (or elements), a **simple arrangement** of p (distinct) objects is any **ordered sequence** of p of these objects. In other words, both choice and order matter.

- This definition implies that:
  - To form an arrangement, we must first choose p objects out of n and then order them (e.g., assign them positions from 1 to p);
  - Two arrangements of p objects can therefore differ either by the chosen objects or by their order.

Notation

We denote by:

$$A_n^p = \frac{n!}{(n-p)!}$$

the number of arrangements of p objects chosen from n distinct objects.

$$A_n^p = \frac{n!}{(n-p)!} = n \times (n-1) \times \cdots \times (n-p+1).$$

Example 1

1) How many arrangements can be formed by taking two objects from a set of four?

- Let the objects be a, b, c, d. By choosing two of them and ordering them, we get:

$$A_4^2 = \frac{4!}{(4-2)!} = 4 \times 3 = 12.$$

- The 12 possible arrangements are:

$$ab,\ ac,\ ad,\ bc,\ bd,\ cd,\ ba,\ ca,\ da,\ cb,\ db,\ dc.$$

Examples 2-3

2) In how many ways can 3 different files be placed in 15 empty lockers, one file per locker?

$$N = A_{15}^{3} = \frac{15!}{(15-3)!} = 2730 \text{ ways.}$$

3) Using the 26 letters of the alphabet, how many 5-letter "words" with distinct letters can be formed?

$$N = A_{26}^{5} = \frac{26!}{(26-5)!}.$$

Remark

- An arrangement of p objects chosen from n can be viewed as a successive selection process:
  - First choose one object from n,
  - then another from the remaining $(n-1)$, and so on.

The order of selection determines the arrangement.

Permutation

- A **permutation** is a special case of an arrangement where $p = n$. Thus, a permutation of n objects is an **ordered sequence of all n objects**.

  - Two permutations of the same n objects differ only in the order of those objects.

Counting Permutations

Number of permutations:

$$P_n = A_n^n = n!.$$

Example: The possible permutations of the letters a, b, c are:

$$abc,\ acb,\ bac,\ bca,\ cab,\ cba.$$

Hence,

$$P_3 = 3! = 6.$$

Arrangements with Repetition

- We now consider a different type of selection process:
  - Each time we select an object, it is returned to the set before the next selection;
  - This process is repeated p times.
- The resulting sequence is called an **arrangement with repetition** of p objects chosen from n.

Counting Arrangements with Repetition

- An arrangement with repetition allows each element to appear more than once, up to p times.

  - The total number of such arrangements is:

$$\alpha_n^p = n^p.$$

Examples

1. The total number of arrangements of order 2 from the letters a, b, c is:

$$\alpha_3^2 = 3^2 = 9.$$

List: $(aa, ab, ac, ba, bb, bc, ca, cb, cc)$.

2. How many numbers can be formed using the digits 1, 2, 3, and 4 (repetition allowed)?

$$\alpha_4^4 = 4^4 = 256.$$

Combination: Definition

- Given a set of n distinct objects, a **combination** of p of these objects is any *unordered* selection of p elements from this set (i.e., order does not matter).

  - Two combinations containing p objects can differ only in the actual objects they include, not in their order.

Counting Formula

Number of combinations:

The total number of possible combinations is given by:

$$C_n^p = \frac{n!}{p!(n-p)!}.$$

- Recall that an arrangement (ordered selection) of p elements among n is given by $A_n^p = \frac{n!}{(n-p)!}$.
- Since each unordered combination of p elements can be arranged in $p!$ different ways, we have:

$$A_n^p = p!\, C_n^p \quad \Rightarrow \quad C_n^p = \frac{A_n^p}{p!}.$$

Example 1

Example:

The combinations of 3 letters that can be formed from the 4 letters A, C, G, and T are:

$$ACG, ACT, AGT, CGT.$$

Hence, there are $C_4^3 = 4$ possible combinations.

Example 2

1. During an oral exam, a student must answer 5 questions out of 8. $\Rightarrow$ Number of possible selections: $C_8^5 = 56$.
2. If the first 3 questions are compulsory: $\Rightarrow$ The student must choose 2 additional questions among the remaining 5: $C_5^2 = 10$.
3. If the student must answer at least 4 of the first 5 questions:

   Case 1: 4 of the first 5, plus 1 among the remaining 3: $C_5^4 \times C_3^1$.

   Case 2: all 5 of the first 5, plus none of the remaining 3: $C_5^5 \times C_3^0$.

   Therefore, total possible choices:

$$C_5^4 \times C_3^1 + C_5^5 \times C_3^0 = 16.$$

Example 3

1. How many groups of six people can be formed from 4 boys and 6 girls if the group must contain:
   - exactly 2 boys: $C_4^2 \times C_6^4$,
   - at least 2 boys: $C_4^2 \times C_6^4 + C_4^3 \times C_6^3 + C_4^4 \times C_6^2$.
2. How many distinct 8-card hands can be drawn from a deck of 32 cards? $C_{32}^8 = 10{,}518{,}300$ possible hands.

Exercise 1

In a sample of **1000 patients**, it was recorded that:

- 300 patients suffer from **lung disease** (P),
- 600 patients suffer from **heart disease** (C), and
- 200 patients suffer from **hypertension** (H).

1. Calculate the number of patients suffering from both **hypertension** and **heart disease**, given that **76% of all patients** suffer from either hypertension or heart disease.

Exercise

2. Given that the number of patients suffering from both **heart disease** and **lung disease** is 60, and that no patient suffers from all three conditions, calculate the number of people suffering from both **hypertension** and **lung disease.**
3. What is the probability of finding a patient suffering from either **hypertension** or **lung disease**?

Exercise 2

A new vaccine was tested on **12500 individuals.** Among them, **75 people**, including **35 pregnant women**, experienced adverse reactions that required hospitalization. Out of the total 12500 participants, **680 were pregnant women.**

1. What is the probability that a **pregnant woman** experiences an adverse reaction after receiving the vaccine?
2. What is the probability that a **non-pregnant person** experiences an adverse reaction?

Exercise 3

A group consisting of **80 men** and **60 women** must select **10 members** to be on duty tonight. If the selection is made at random, what is the probability that the duty team:

$a)$ consists **only of men**?

$b)$ consists **only of women**?

$c)$ consists of an **equal number of men and women**?

Exercise 4

Eighteen people volunteered for a blood donation drive. Among them, there were: 11 individuals with blood group **O**, 4 with group **A**, 2 with group **B**, and 1 with group **AB**. After the collection, **three blood samples** are randomly selected from the 18 collected units.

Compute the probability of each of the following events:

1. The three selected blood samples belong to the **same blood group**;
2. Among the three selected samples, there is **at least one sample of blood group A**;
3. The three selected blood samples belong to **three different blood groups**.

Exercise 5

Treating a patient requires the administration of **two different syrups** and **three different types of tablets.** The doctor has access to **three kinds of syrups** and **four kinds of tablets**, all of which have similar therapeutic effects.

How many distinct prescriptions can the doctor write, given that **one specific syrup must not be prescribed together with one specific type of tablet**?

Exercise 6

In a city, there are three emergency rescue centers. Five patients each make a phone call on the same day, choosing one of the centers at random from the phone directory.

What is the probability that **all three centers** receive at least one call?

Exercise 7

DNA is composed of four bases: **A, C, G, T**. An amino acid is encoded by a sequence (or "word") of three letters formed from this alphabet.

1) How many different amino acid codons can be formed?
2) What is the probability that an amino acid contains **exactly two distinct letters**?

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