Probability Tutorials

Exercice 1 *In an animal pharmacology experiment, 2 tablets are randomly selected from an opaque batch containing 7 indistinguishable tablets. Among these tablets:*

- *1 contains no active ingredient (placebo),*
- *2 contain 100 mg of active ingredient,*
- *2 contain 200 mg,*
- *2 contain 300 mg.*

The 2 selected tablets are administered to a given animal, and we consider the random variable X as the total dose ingested by the animal.

1. *Calculate the probability distribution of X.*
2. *Calculate the expectation and variance of X.*

Exercice 2 *The age X at which a childhood disease appears follows a normal distribution with mean 3 years and standard deviation 1 year.*
Calculate the probabilities of the disease appearing:

- *Before 6 months,*
- *Between 1 and 2 years,*
- *After 6 years.*

Exercice 3 *The time needed by a student to complete an exam is modeled as a normal variable with mean $\mu = 90$ minutes and standard deviation $\sigma^2 = 45$ minutes. 240 students take this exam.*

1. *How many students will finish the exam in less than 2 hours?*
2. *What should be the exam duration so that 200 students can finish it?*

Exercice 4 *We studied the blood glucose levels in a population of individuals with specific characteristics and obtained:*

- *20% of glucose levels are below 0.82 g/L,*
- *30% are above 0.98 g/L.*

Assuming blood glucose follows a normal distribution, determine the mean and standard deviation of this distribution.

Exercice 5 *Consider a group of 4 individuals selected from the Algerian population aged 60 to 75 years. The number of people with hypertension in this sample is a binomial random variable with parameters $n = 4$ and $p = 0.15$.*

1. *What are the probabilities that 0, 1, 2, 3, or 4 individuals have hypertension?*
2. *Represent the probability distribution of this variable.*
3. *Represent its cumulative distribution function (CDF).*

Exercice 6 *It is known that the probability that a person is allergic to a certain medication is 10^{-3}.*
Consider a sample of 1,000 people. Let X be the random variable representing the number of allergic individuals in the sample.

1. *Determine, with justification, the probability distribution of X.*

2. *Using an appropriate approximation (to be justified), calculate the probabilities of the following events:*

   (a) Exactly two people in the sample are allergic.

   (b) At least two people in the sample are allergic.

Exercice 7 *The emergency department of a hospital receives, on average, two cases every 15 minutes. Calculate the probabilities that the number of emergency cases in one hour is:*

- *Exactly 7,*
- *Greater than 7.*

Exercice 8 *Suppose that the number of white blood cells per unit volume in a diluted blood sample (counted using a microscope) follows a Poisson distribution with mean 100. Calculate the probability of observing at least 90 white blood cells in the next measurement.*

Exercice 9 *It is estimated that the probability that a person of vaccination age requests a flu vaccine is 0.4. For a population of 20,000 individuals of vaccination age, determine the number of vaccines that must be available so that the probability of running out is less than 0.1.*