Probability Tutorials

Exercice 1 *A laboratory contains a cage with 100 mice exhibiting two characteristics: sex (male or female) and color (white or black). Among them, 87 are male, 57 are white, and 55 are both male and white.*

1. *Determine the number of mice in each category.*
2. *An assistant selects one mouse at random. Compute the probability that the selected mouse is **white** or **male**.*
3. *She decides to select 6 mice. Compute the probability that all 6 selected mice are white if the selections are made:*
   (a) *with replacement;*
   (b) *without replacement.*

Exercice 2 *To inspect a batch of medication (amoxicillin), 5 tablets are drawn simultaneously from a jar containing 40 tablets of identical size but of different colors: 12 red, 8 yellow, 10 white, and 10 gray.*
Compute the probability that:

- *all 5 selected tablets are **yellow**;*
- *at least one selected tablet is **red**.*

Exercice 3 *Three medical residents, A, B, and C, take the DEMS examination. Any number of them (one, two, or all three) may fail. The probabilities that each resident fails the exam are respectively* 0.12, 0.25, *and* 0.30.
*Using two different methods, compute the probability that there is **no total failure** (i.e., not all three fail simultaneously).*

Exercice 4 *In a population of patients presenting abdominal pain,* 30% *have acute appendicitis. Among those with appendicitis,* 70% *have a body temperature above* $37.5°C$. *Among patients without appendicitis, a temperature above* $37.5°C$ *occurs in* 40% *of cases.*
For a randomly selected patient, define the events:
A: *the patient has acute appendicitis;* T: *the patient has a body temperature above* $37.5°C$.

1. *Compute the following probabilities:* $\Pr(A)$, $\Pr(T \mid A)$, $\Pr(T \mid \overline{A})$, $\Pr(\overline{T} \mid A)$, *and* $\Pr(T)$.
2. *For a patient with a temperature above* $37.5°C$, *compute the probability **P1** that they have acute appendicitis.*
3. *For a patient with a temperature not exceeding* $37.5°C$, *compute the probability **P2** that they do not have acute appendicitis.*

Exercice 5 *One quarter of a population has been vaccinated against a contagious disease. During an epidemic, it is observed that among the sick individuals, there is* one vaccinated person for four unvaccinated ones. *It is also known that during this epidemic,* one vaccinated individual out of twelve *becomes sick.*
A person is chosen at random from this population.

1. *The probability of being unvaccinated given that the person is sick is:*
   $(A)\ \frac{1}{5}$ $(B)\ \frac{4}{5}$ $(C)\ \frac{1}{12}$ $(D)\ \frac{1}{48}$ $(E)\ \frac{1}{9}$
2. *The probability of being sick is:*
   $(A)\ \frac{1}{12}$ $(B)\ \frac{1}{4}$ $(C)\ \frac{1}{48}$ $(D)\ \frac{5}{48}$ $(E)\ \frac{149}{240}$
3. *The probability of being healthy and vaccinated is:*
   $(A)\ \frac{11}{48}$ $(B)\ \frac{1}{16}$ $(C)\ \frac{1}{9}$ $(D)\ \frac{1}{4}$ $(E)\ < 0.02$
4. *The probability of being sick given that the person is unvaccinated is:*
   $(A)\ > 0.7$ $(B)\ \frac{1}{5}$ $(C)\ \frac{4}{5}$ $(D)\ \frac{1}{9}$ $(E)\ \frac{149}{240}$

5. *The vaccine is:*
   (A) *effective* (B) *not effective* (C) *independent of the disease* (D) *unrelated to the disease* (E) *cannot be determined*

Exercice 6 *Plant tissue cultures may be infected either by fungi or by bacteria. The probability of fungal infection is* 15%*, and the probability of bacterial infection is* 8%*.*

1. *Compute the probability of a simultaneous infection by both fungi and bacteria:*
   - *assuming the infections are independent;*
   - *assuming the infections are not independent and that the probability of bacterial infection given fungal infection is* 4%*.*
2. *Compute the probability of being infected (from any source) in both cases above.*

Exercice 7 *All numerical answers must be approximated to four decimal places.*

You have just taken a screening test for a certain type of cancer. The doctor calls you in and announces the result: bad news, your test is positive. This type of cancer affects only 0.1% *of the population.*

You ask the doctor whether the test is reliable. He responds clearly: "If you have the cancer, the test is positive in 90% *of cases"; "if you do not have it, the test is negative in* 97% *of cases".*

1. *The probability of having a positive test given the presence of cancer is:*
   (A) 0.0009 (B) 0.001 (C) 0.03 (D) 0.9 (E) 0.97
2. *The probability of being a person with cancer* and *testing positive is:*
   (A) 0.0009 (B) 0.001 (C) 0.03 (D) 0.9 (E) 0.97
3. *The probability of testing positive for a healthy person is:*
   (A) 0.0009 (B) 0.001 (C) 0.03 (D) 0.0309 (E) 0.9691
4. *The probability of having a negative test result in the population is:*
   (A) 0.0009 (B) 0.009 (C) 0.03 (D) 0.9 (E) 0.9691
5. *After receiving such a test result, what is the probability that you actually have cancer?*
   (A) $> 90\%$ (B) $= 90\%$ (C) $= 9\%$ (D) $= 5\%$ (E) $< 3\%$

$\mathscr{B}.\ \mathscr{T}.$