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Dynamique des gaz

Cours et Travaux dirigés

« Gas dynamics »

Cours destiné aux étudiants :

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Preamble

This document regarding gas dynamics adheres to the official framework established by the Ministry of Higher Education and Scientific Research MESRS. It is designed for first-year students enrolled in the Master1 program in **Energetics** within the field of **Mechanical Engineering**.

The dynamics of gases is a branch of fluid mechanics that examines the behavior of gases in motion. It plays a significant role in various fields of science and engineering, ranging from the aerodynamics of aircraft to the flow in jet engines, as well as atmospheric flows, explosions, and even space propulsion.

Unlike liquids, gases are compressible, meaning their density can vary significantly under the influence of pressure and temperature. This compressibility is crucial for describing dynamic phenomena, particularly in supersonic regimes where shock waves and marked discontinuities in flow occur.

The dynamics of gases is based on the application of fundamental laws of physics: conservation of mass, momentum, and energy, which are typically expressed in the Navier-Stokes equations adapted for compressible gases. In non-viscous and adiabatic conditions, the Euler equations are specifically employed, coupled with an equation of state (usually that of ideal gases).

Understanding gas dynamics is vital for designing efficient and safe systems in contexts where speed, pressure, and temperature fluctuate rapidly. It is also a field where physical phenomena and mathematical tools are closely intertwined, requiring rigorous modeling and often advanced numerical simulations.

A gas dynamics is the field of dynamics that deals with the movement of air and other gaseous fluids, as well as the forces acting on a body in relative motion to these fluids. Such compressible flows are found in pipelines transporting natural gas, or through the diffuser of an aircraft's turbojet, within turbines and compressors. Likely, the two most significant effects of flow compressibility are:

1. Suffocation: where the flow velocity within the conduit (internal) is strictly limited by the sonic condition.
2. Shock waves: which represent minor discontinuities in the characteristics of supersonic flow.
3. The flows in conduits with a constant cross-section are analyzed in both theories, namely with friction and without heat transfer, referred to as the FANNO theory, and the other without friction and with heat transfer, known as the Rayleigh theory.

Summary

CHAPTER I: Introduction to gas dynamics

I.1 Concepts and relations in thermodynamics

The dynamics of compressible fluids examines the laws governing the movement of compressible fluids at high flow velocities comparable to the speed of sound. Generally, one studies one-dimensional flows.

The comprehensive study of compressible fluid flows cannot be conducted without incorporating thermodynamics.

The analysis of compressible fluid flow cannot be undertaken without first establishing a number of simplifying assumptions (nature of the gas: ideal, type of process: isothermal or adiabatic, etc.).

We will limit our study to the case of an ideal, non-viscous fluid in steady one-dimensional flow. To clarify:

Ideal fluid = perfect gas (P v = r T) at constant g

Steady flow: time-independent

One-dimensional: all quantities depend solely on one spatial parameter, the curvilinear abscissa, and they remain constant in a section perpendicular to the mean line.

Most of the time, a perfect gas is regarded as a fluid.

A gas whose state parameters change according to the equation.

$\frac{P}{\rho} = r\,T$ is referred to as an ideal gas.

In thermodynamic terms, $r = \frac{R}{\mathcal{M}} = \frac{83{,}14\, kj/k \quad .k}{molar\ mass(kg/kmol)} = \frac{83{,}14\,.10^3}{28{,}97} = 287\, j/kg.k$

According to Mayer's account:

$r = c_p - c_v$ and consequently $c_v = \frac{r}{\gamma - 1}$ et $c_p = \frac{\gamma\, r}{\gamma - 1}$ with $\gamma = \frac{c_p}{c_v}$

Adiabatic exponent:

If $\gamma = cte$, the gas is ideal in terms of calorific value.

Each state of gas corresponds to the state functions.

Internal energy:$U = c_v.T$

Enthalpy :$H = c_p\,T$

This sum is referred to as "total enthalpy." It remains conserved even when the flow is irreversible (with friction).

Entropy :

$$S = c_v \, lnPv^{\gamma} + cte \tag{I.1}$$

The state of gas can only be altered through interactions with the surrounding environment. The energy equilibrium equation of an open system is:

$$\Delta E_c + \Delta E_P + \Delta H \quad = Q \quad + W_u \tag{I.2}$$

ΔE_c: Variation of kinetic energy.

ΔE_P: Variation of potential energy.

ΔH : Variation of enthalpy.

Q : Amount of heat provided by the external environment

W_u : Total work provided by external forces

It is assumed:

There is no exchange of useful work., $W_u = 0$;

The potential energy is negligible, $\Delta E_P = 0$;

The flow is adiabatic and reversible, $Q \quad = 0$;

The energy balance equation becomes: $\Delta E_c + \Delta H \quad = 0$

Or else:

$$(H_2 - H_1) + \frac{1}{2}(C_2^2 - C_1^2) = 0 \tag{I.3}$$

Therefore:

$$H + \frac{1}{2}C^2 = Cte \tag{I.4}$$

I.2 Isentropic relations of an ideal gas

According to the first principle of thermodynamics:

$-Pv^{\gamma} = cte$ which assumes a specific value of n for each transformation to be studied.

For the reversible adiabatic transformation, $n = \gamma \Rightarrow Pv^{\gamma} = cte$:The Poisson equation.

For such a transformation:

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{I.5}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} , dq = 0, dh = -\gamma \, P \, dv = vdP \tag{I.6}$$

For a reversible isothermal transformation n=1⇒ $Pv = cte$ the equation of Mariotte

The second principle of thermodynamics states that:

$$dS \geq \frac{dq}{T} \qquad \text{(I.7)}$$

Equality is relative to reversible transformations, without thermal exchanges. $dq = 0 \Rightarrow dS = 0, S_2 = S_1$

It is said that the adiabatic and reversible transformation is isentropic.

The irreversible adiabatic transformation is not isentropic; for such a transformation. $dS > 0 \Rightarrow S_2 > S_1$

One of the most significant causes of irreversibility is associated with the presence of frictional forces due to viscosity. A fluid is considered perfect in the aerodynamic sense if it is devoid of "viscosity".

I.3 Compressibility and propagation of sound waves

The compressibility coefficient is defined by the following relationship:

$$\chi_{index} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right) = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial \rho}\right)_{index} \qquad \text{(I.8)}$$

In the event of an isothermal transformation: $P\, v^{\gamma} = cte$

$$Pdv + vdP = 0, \qquad \frac{dv}{v} + \frac{dP}{P} = 0 \Rightarrow \left(\frac{\partial v}{\partial P}\right)_T = -\frac{v}{P} \qquad \text{(I.9)}$$

And

$$\chi_T = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{P} \qquad \text{(I.10)}$$

For an isentropic transformation

$$P\, v^{\gamma} = cte \Rightarrow \frac{dP}{P} + \gamma\frac{dv}{v} = 0 \Rightarrow \left(\frac{\partial v}{\partial P}\right)_S = -\frac{1}{\gamma}\,\frac{v}{P} \qquad \text{(I.11)}$$

And

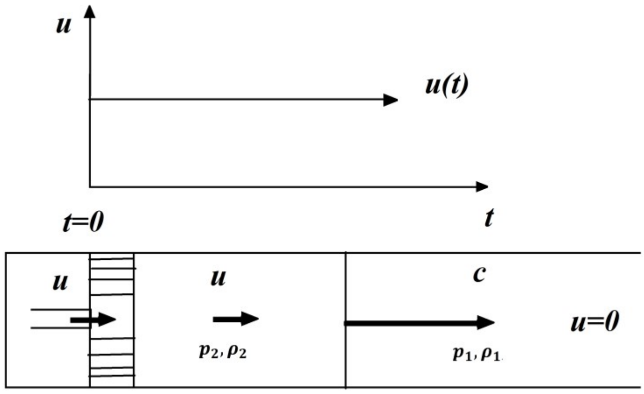
$$\chi_S = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_S = \frac{1}{\gamma P} \qquad \text{(I.12)}$$

It is observed that $\frac{\mathcal{X}_T}{\mathcal{X}_S} = \gamma$ ou $\mathcal{X}_T = \gamma \mathcal{X}_S$ (I.13)

It is observed that $\mathcal{X}_T > \mathcal{X}_S$

I.4 General Expression of the speed of sound

By definition, the speed of sound refers to the rate at which a pressure disturbance propagates through a compressible fluid.



In the fluid, a shock wave is observed propagating in front of the piston at a speed of $C < U$. The shock wave sets the fluid in motion such that a particle located upstream of the wave remains at rest in a fixed reference frame relative to the tube. The event is unsteady. To determine the speed of the wave, a reference frame relative to it is selected. In such a reference frame, the flow is steady.

L'application de la deuxième loi de Newton au volume de contrôle de la figure précédente, donne (F=0) d'où :

$$(P_1 - P_2)A = \rho_2\, A(C - U)^2 - \rho_1 A\, C^2 \quad \text{(I.14)}$$

The continuity equation, which requires a constant mass flow rate, is expressed as:

$$\rho_1\, A\, C = \rho_2 A\, (C - U) \quad \text{(I.15)}$$

It follows that:

$$P_1 - P_2 = \frac{\rho_1^2 C^2}{\rho_2} - \rho_1 C^2 = \rho_1 \left(\frac{\rho_1}{\rho_2} - 1\right) C^2 \quad \text{(I.16)}$$

$$C = \left[\frac{P_2 - P_1}{\rho_2 - \rho_1}\frac{\rho_2}{\rho_1}\right]^{1/2} \tag{I.17}$$

$$[C^2] = \frac{N}{m^2}\;\frac{m^3}{kg} = \left(\frac{m}{s}\right)^2$$

The acoustic wave is characterized by a low speed U in relation to C and a slight disturbance of ρ, $\rho_1 \simeq \rho_2$. In this case, the speed C of the wave approaches the acoustic speed a.

$$C = \left[\frac{\Delta P}{\Delta \rho}\right]^{1/2} = a \tag{I.18}$$

Consequently, the fluid movement associated with an acoustic wave is isentropic. Thus, the relationship between pressure and density in an acoustic wave is also isentropic. Therefore, the speed of sound is defined by:

$$\boxed{a^2 = \left(\frac{\partial P}{\partial \rho}\right)_S = \gamma\frac{P}{\rho} = \gamma\, r\, T} \tag{I.19}$$

I.5 Mach number and Mach waves

The flow similarity parameter is the Mach number.

The Mach number changes from one point to another in the flow, not only due to variations in speed but also because the state of the fluid changes, thus affecting the speed of sound.

C: Velocity of the gas movement

a: The speed of sound, also referred to as the speed of sound, is expressed by the following equation:

$$a = \sqrt{\gamma\, r\, T} \tag{I.20}$$

Let us consider a mobile object moving from right to left at a speed of c, thus achieving a Mach number

$$M = \frac{C}{a} \tag{I.21}$$

I.6 Subsonic, transonic, supersonic, and hypersonic flow

if : $M = 1$: The flow velocity is **sonic**.

if: $M < 1$: The flow is referred to as **subsonic** if the flow velocity is less than the speed of sound.

near the speed of sound is **transonic** $\boldsymbol{M \leqq 1}$

if : $M > 1$: The flow is referred to as **supersonic** if the flow velocity exceeds the speed of sound.

And $M \ggg 1$: The flow regime is very much greater than the speed of sound is **hypersonic**.

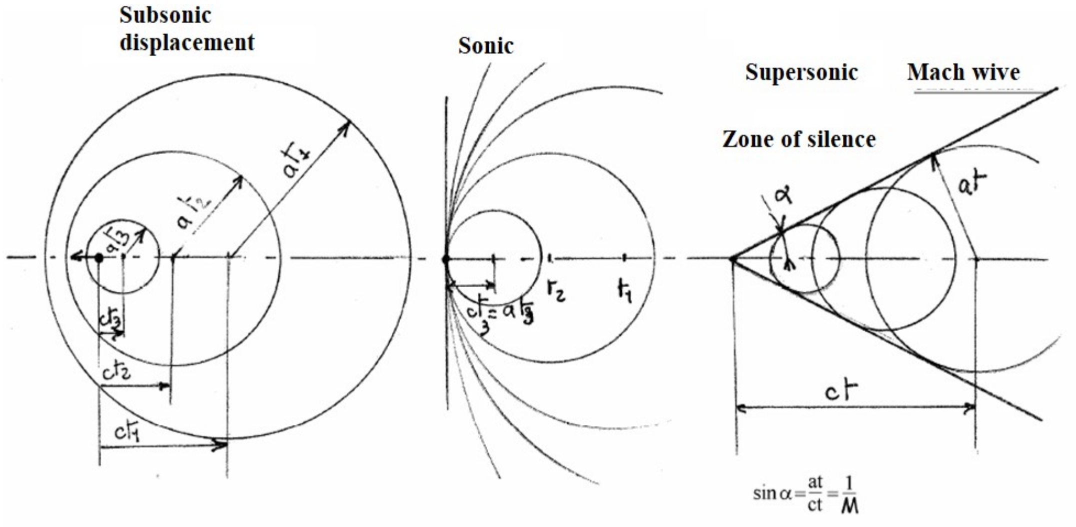


Figure I.1 Acoustic Mach wave

In subsonic conditions, the fluid upstream of the moving object receives pressure waves prior to the object's arrival, allowing it to continuously deviate as the object approaches.

In supersonic flow, the fluid upstream of the sonic cone is not forewarned of the approach (silent zone) and can therefore only deviate abruptly (shock) upon the passage of the wave: this is the fundamental difference between the two types of flow.

Example:

An aircraft flies at a speed of 400 m/s. Calculate the Mach number for:

a) The standard condition at sea level (T=289 K)

b) At an altitude of 15200 m, under standard conditions (T=217 K)

Solution :

The number of Mach $M = \frac{C}{a} = \frac{C}{\sqrt{\gamma\, r\, T}}$

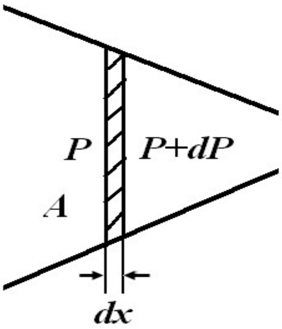
a) $M_1 = \frac{C}{\sqrt{\gamma\, r\, T_1}} = \frac{400}{\sqrt{(1.4)(287)(289)}} = 1.17$, b) $M_2 = \frac{C}{\sqrt{\gamma\, r\, T_2}} = \frac{400}{\sqrt{(1.4)(287)(217)}} = 1.35$

CHAPTER II: Unidimensional isentropic flow in a variable cross-section duct

II.1 Basic Equation (Continuity, Momentum, Energy)

II.1.1 Equation of mass conservation (Continuity)

$$\dot{m} = C.\rho.A \qquad \left({}^{kg}/_{s}\right) \qquad \text{(II.1)}$$



$\dot{m}$: Mass flow rate through a given section A, for steady flow.

$m \dot{=} cte$, $\dot{m} = \frac{A.C}{v}$, $\dot{m}.v = A.C$

$\frac{\dot{m}}{A} = \rho.C = g_m$: Specific mass flow rate.

If A=cte therefore $g_m = cte$ That is to say : $\rho.C = cte$

II.2 Equation of motion

S According to Newton's

$$F = \underbrace{m}_{mass} \cdot \underbrace{\frac{dC}{dt}}_{acceleration} \qquad \text{(II.2)}$$

$$\underbrace{(P - P - dP).A}_{the\ force} = \underbrace{\rho.A.dx}_{mass}.\underbrace{\frac{dc}{dt}}_{acceleration} \qquad \text{(II.3)}$$

With : $\frac{dx}{dt} = C$ We possess: $-dP = \rho.d\left(\frac{C^2}{2}\right)$ (II.4)

Let it be :

$$-v\,dP = d\left(\frac{C^2}{2}\right) \qquad \text{(II.5)}$$

II.1.3 Energy equation

According to the first principle of thermodynamics $dq = dh - v\,dP$ However $-v\,dP = d\left(\frac{C^2}{2}\right)$

$\Rightarrow d\left(\frac{C^2}{2}\right) + dh = 0$ Or in its integral form $\frac{C^2}{2} + h = cte$: equation of energy conservation

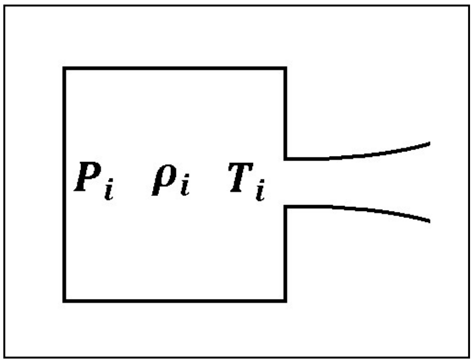
It is known that $h = c_p.T$ However $c_p = \frac{\gamma}{\gamma-1}\,r$

$$\Rightarrow \frac{C^2}{2} + \frac{\gamma}{\gamma - 1}\,r.T = cte \qquad \text{(II.6)}$$

$$\Rightarrow \frac{C^2}{2} + \frac{\gamma}{\gamma - 1}\,\frac{P}{\rho} = cte \qquad \text{(II.7)}$$

II.2 General Laws of isentropic flow: generating state and critical state

Stagnation parameters and generator parameters

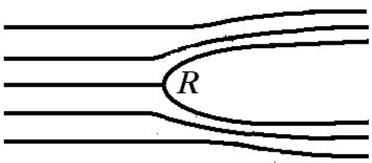


This term refers to the state of the fluid at a point in the flow where the velocity is assumed to be zero; it is characterized by the index ***i***.

The features within this reservoir are those of the generating state; this representation justifies the term "generating state".

II.2.1 Stagnation state

Let there be a stagnation point defined in the following figure:



$$\frac{C^2}{2} + h = cte \ = h_0 = h_i = c_p T_0 \qquad \text{(II.8)}$$

$$\Rightarrow T_0 = T + \frac{C^2}{2c_p} \qquad \rightarrow \text{Stagnation temperature} \qquad \text{(II.9)}$$

It is the temperature that will be measured with a thermometer placed in the flow.

As $h_0 =$ cte therefore $T_0 = cte$ (identical in each section of the flow).

h_0 : Stagnation enthalpy.

$$\frac{T_0}{T} = 1 + \frac{C^2}{2h} \ \text{more} \qquad h = c_p.T = \frac{\gamma}{\gamma-1}\, r \frac{a^2}{\gamma\, r} = \frac{a}{\gamma-1} \qquad \text{(II.10)}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2 \qquad \text{(II.11)}$$

$$\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/_{\gamma-1}} \qquad \text{(II.12)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/_{\gamma-1}} \qquad \text{(II.13)}$$

In the event that gas escapes from the center where the initial velocity C_0 is negligible compared to the speed in the section considered *C*, in a:

$$\frac{C^2}{2} - \frac{C_i^2}{2} = h_i - h \quad \Rightarrow C = \sqrt{2(h_i - h)} \qquad \text{(II.14)}$$

h_i : Generator enthalpy, $h_i = c_p\, T_i$ avec T_i : generator temperature.

$T_i = \frac{P_i}{\rho_i .r}$ ou P_i : generator temperature, ρ_i : generator density.

- - As $\frac{C^2}{2} + h = h_i$, It is observed that: $h_i = h_0$ and $T_i = T_0$
- In an adiabatic flow, the stagnation enthalpy and stagnation temperature correspond to the generating enthalpy and generating temperature; therefore,:

  $P_i = P_0$ and $\rho_i = \rho_0$, It is therefore written.:

$$\frac{T_i}{T} = 1 + \frac{\gamma-1}{2} M^2 \qquad \text{(II.15)}$$

$$\frac{P_i}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma - 1} \quad \text{(II.16)}$$

$$\frac{\rho_i}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/\gamma - 1} \quad \text{(II.17)}$$

II.2.2 Comparison with incompressible flow

$$P_0 - P = \rho \, \frac{C^2}{2} \qquad \text{where} \qquad \frac{P_0 - P}{\rho \, C^2/_2} = 1 \quad \text{(II.18)}$$

For a compressible fluid in subsonic isentropic flow:

$$\frac{P_i}{P} = \left(1 + \frac{(\gamma - 1)\, M^2}{2}\right)^{\frac{\gamma}{\gamma - 1}} \quad \text{(II.19)}$$

$$\frac{P_i}{P} = 1 + \frac{\gamma}{2} M^2 + \frac{\gamma}{8} M^4 + \frac{\gamma(2-\gamma)}{48} M^6 + \ldots\ldots\ldots$$

$$P_i - P = \frac{P\gamma\, M^2}{2}\left(1 + \frac{M^2}{4} + \frac{2-\gamma}{24} M^4\right)$$

However: $\frac{P\gamma\, M^2}{2} = \frac{P\gamma\, C^2}{2\,\gamma\, rT} = \frac{\rho\, C^2}{2}$

From where :

$$\frac{P_i - P}{\rho\, C^2/_2} = 1 + \frac{M^2}{4} + \frac{M^4}{40} + \frac{M^6}{1600} + \ldots.. \quad \text{(II.20)}$$

Hence the following table:

M	0.1	0.2	0.3	0.4	0.5
$\frac{P_i - P}{\rho\, C^2/_2}$	1.003	1.010	1.023	1.041	1.064

At low speeds when $\frac{M^2}{4} < 1$ The Bernoulli formula has been modified. :

$$\frac{P_i - P}{\rho\, C^2/_2} = 1 \quad \text{(II.21)}$$

$$P_i - P \; = \rho\, C^2/_2 \quad \text{and} \quad P_0 - P \; = \rho\, C^2/_2 \quad \text{(II.22)}$$

$\Rightarrow P_0 = P_i = P \quad + {}^{\rho\, C^2}/_2$ Bernoulli's equation for incompressible fluid.

${}^{\rho\, C^2}/_2$: Dynamic pressure

As $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}}$ si $\frac{\gamma-1}{2} M^2 < 1$

$\frac{\rho_0}{\rho} \approx 1 + \frac{M^2}{2} + \cdots$ The relative variation of density between the upstream state and any given section.

$$\frac{\rho_0}{\rho} - 1 = \frac{\rho_0 - \rho}{\rho} = \frac{M^2}{2} = \frac{\rho_i - \rho}{\rho} \qquad \text{(II.23)}$$

for:

M=0.1 $\quad \frac{\rho_0-\rho}{\rho} = 0.5\%$

M=0.14 $\quad \frac{\rho_0-\rho}{\rho} = 1\%$

M=0.2 $\quad \frac{\rho_0-\rho}{\rho} = 2\%$

For the air at 15°C and 340m/s et $\frac{\rho_0-\rho}{\rho} = 1\%$ corresponds to C=47.6m/s and ***The assimilation of air as an incompressible fluid is justified in low-speed domains..***

II.2.3 Barré-Saint Venant formula

Given the result concerning the speed calculation in the form of,

$$C = \sqrt{2(h_0 - h\ \ )} \qquad \text{(II.24)}$$

One can provide him with an expression referred to as the "Barré-Saint formula".

$$C = \sqrt{2\, cp\, (T_0 - T\ \ )} = \sqrt{2\, cpT_0 \left(1 - \frac{T}{T_0}\right)}$$

$$C \ = \sqrt{2\, \frac{\gamma}{\gamma-1}} \qquad \text{(II.26)}$$

$C = \sqrt{\frac{2\gamma}{\gamma-1} \frac{P_0}{\rho} (1 - \left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}})}$ ***Barré-Saint Venant formula***

II.2.4 Speed limit

It is evident that the increase in speed occurs at the expense of pressure, or rather, due to the expansion of the gas. If the expansion were to be pushed to the point of a vacuum (a hypothetical situation), a limiting speed would be reached.

$$C = \sqrt{\frac{2\gamma}{\gamma-1}\frac{P_0}{\rho}} = \sqrt{\frac{2\gamma\, r\, T_0}{\gamma-1}} \tag{II.27}$$

By designating $a_0^2 = \gamma\, r\, T_0$ We write $C_{lim}=a_0\sqrt{\frac{2}{\gamma-1}}$

AN : This speed is noteworthy for several reasons :

- The speed mentioned is already the maximum velocity that exhaust gases from a rocket can achieve in a vacuum; thus, it determines the maximum thrust of rocket engines.
- Furthermore, from the perspective of kinetic gas theory, this speed signifies the disordered velocity of molecules within the gas. Indeed, when the gas is expanded without energy loss to a state of zero pressure and temperature, the kinetic energy of the molecules relative to one another becomes zero. The kinetic energy of all the molecules fixed in relation to each other is equal to the energy prior to expansion, which corresponds to the disordered kinetic energy of the molecules in the gas (assuming ideal gas conditions).

It is easy to verify that for the air taken at $T_0 = 15°C$, on a T_0=288k

$a_0=\sqrt{(1.4)\,.\,(287).(288)}$=340m/s and $C_{lim} = 340\sqrt{\frac{2}{1.4-1}}$=760m/s

II.2.5 Critical parameters

When gas expansion occurs in a pipeline, it is possible that in a section referred to as 'critical', the flow velocity becomes equal to the local speed of sound. This velocity, also known as 'critical', is observed in this section.

$$M = 1 \Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2 \tag{II.28}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}(1)^2$$

$$\Rightarrow T = T_C = \frac{2}{\gamma + 1}T_0 \tag{II.30}$$

with M=1 provides us with the so-called critical temperature.

$$\frac{P_C}{P} = \left(\frac{2}{\gamma + 1}\right)^{\gamma/\gamma-1} \quad \text{(II.31)}$$

For the critical

$$C_c \;= a_c \;= \sqrt{\gamma \;\; r \;\; T_c} \;= \sqrt{\frac{2\,\gamma}{\gamma+1} r \;\; T_0} \;= \sqrt{\frac{2\,\gamma}{\gamma+1}\frac{P_0}{\rho_0}} \quad \text{(II.32)}$$

The critical speed is a function of the nature of the gas and the generating (or stagnation) conditions.

Same $a_0 = \sqrt{\gamma \;\; r \;\; T_0}$, It is observed that:

$$C_c \;= a_c \;= \sqrt{\frac{2\,\gamma}{\gamma+1}\frac{P_0}{\rho_0}} \;= \sqrt{\frac{2 \;\; \gamma\, r\, T_0}{\gamma+1}} \;= a_0\sqrt{\frac{2}{\gamma+1}} \quad \text{(II.33)}$$

II.2.6 Reduced speed

The relationship between flow velocity and critical velocity is referred to as "reduced velocity.

$$\lambda = \frac{C}{a_c} \quad \text{(II.34)}$$

In the critical section, we simultaneously

$C = a_c \Rightarrow \lambda = 1 \; et \; M = 1$ However, in general :

$$\lambda = \frac{C}{a_c} = \frac{C}{a}.\frac{a}{a_c} = M.\frac{a}{a_c}$$

$$\Rightarrow M = \lambda\,\frac{a_c}{a} = \lambda\,.\frac{\sqrt{\frac{2\,\gamma}{\gamma+1}\frac{P_0}{\rho_0}}}{\sqrt{\gamma\, r\, T}} = \lambda.\sqrt{\frac{\frac{2\,\gamma}{\gamma+1}\, r\; T_0}{\gamma\, r\, T}} = \lambda.\sqrt{\frac{\frac{2\,\not\gamma}{\gamma+1}\not r\, T_0}{\not\gamma \not r\, T}} \quad \text{(II.35)}$$

M= $\lambda.\sqrt{\frac{2}{\gamma+1}\frac{T_0}{T}} = \lambda.\sqrt{\frac{2}{\gamma+1}}.\sqrt{\frac{T_0}{T}}$ this relationship allows for writing with:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad \text{(II.36)}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{\gamma - 1}{2}\left(\lambda^2.\frac{2}{\gamma + 1}.\frac{T_0}{T}\right)$$

$$\frac{T_0}{T} - \left(\frac{\gamma - 1}{2}.\frac{2}{\gamma + 1}.\lambda^2\right)\frac{T_0}{T} = 1$$

$$\frac{T_0}{T}\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)=1$$

$$\Rightarrow \frac{T}{T_0}=1-\frac{\gamma-1}{\gamma+1}\lambda^2 \qquad (II.37)$$

Et

$$\frac{M}{\lambda}=\sqrt{\frac{2}{\gamma+1}}\cdot\sqrt{\frac{T_0}{T}}=\frac{\sqrt{\frac{2}{\gamma+1}}}{^{-1}\sqrt{\frac{T_0}{T}}}=\frac{\sqrt{\frac{2}{\gamma+1}}}{\sqrt{\frac{T}{T_0}}}=\frac{\sqrt{\frac{2}{\gamma+1}}}{\sqrt{1-\frac{\gamma-1}{\gamma+1}\lambda^2}}=\frac{1}{\sqrt{(1-\frac{\gamma-1}{\gamma+1}\lambda^2)\quad\frac{\gamma+1}{2}}} \qquad (II.38)$$

$$\frac{M}{\lambda}=\frac{1}{\sqrt{(\frac{\gamma+1}{\gamma+1}-\frac{\gamma-1}{\gamma+1}\lambda^2)\quad\frac{\gamma+1}{2}}}$$

$$\frac{M}{\lambda}=\frac{1}{\sqrt{\frac{(\gamma+1)-(\gamma-1)}{2}\lambda^2}}=\sqrt{\frac{2}{(\gamma+1)-(\gamma-1)\lambda^2}} \qquad (II.39)$$

M= $\lambda.\sqrt{\frac{2}{(\gamma+1)-(\gamma-1)\lambda^2}}$

At the same time

$$\frac{P}{P_0}=\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{\frac{\gamma}{\gamma-1}} \text{ et } \frac{\rho}{\rho_0}=\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{\frac{1}{\gamma-1}} \qquad (II.40)$$

Therefore, regarding the speed limit (P=0)

$$\left(1-\frac{\gamma-1}{\gamma+1}\lambda^2\right)^{\frac{\gamma}{\gamma-1}}=0 \qquad (II.41)$$

$$\Rightarrow \lambda_{lim}^2\ .\frac{\gamma-1}{\gamma+1}=1$$

$$\Rightarrow \lambda_{lim}=\sqrt{\frac{\gamma+1}{\gamma-1}} \qquad (II.42)$$

if $\gamma=1.4\ \Rightarrow\ \lambda_{lim}=2.45$

$\lambda\ =\frac{C}{a_c}$ In the critical section $\lambda\ =1$

$$\frac{T_c}{T_0}=1-\frac{\gamma-1}{\gamma+1}\lambda^2 \quad \text{with} \quad \lambda^2=1 \qquad \frac{T_c}{T_0}=1-\frac{\gamma-1}{\gamma+1} \qquad (II.43)$$

$$\frac{T_c}{T_0} = \frac{\gamma+1-\gamma+1}{\gamma+1} = \frac{2}{\gamma+1}, \qquad \frac{P_c}{P_0} = \left(\frac{T_c}{T_0}\right)^{\gamma/\gamma-1} = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1}, \tag{II.44}$$

$$\frac{\rho_c}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{1/\gamma-1} \tag{II.45}$$

The gas parameters in the critical section are functions of the nature of the gas and the generating parameters.

with $\gamma = 1.4$ we finds: $\frac{T_c}{T_0} = 0{,}83$; $\frac{P_c}{P_0} = 0{,}528$; $\frac{\rho_c}{\rho_0} = 0{,}634$; $\frac{a_c}{a_0} = 0{,}913$

II. 3 One-dimensional flow in a variable cross-section conduit and Hugoniot's theorem

Reduced flow, mass flow in steady state

$\dot{m} = \rho \quad C \quad A \quad = \rho_c\, C_c A_c$: Section evaluation

$q = \frac{\rho . C}{\rho_c\, C_c} = \frac{A_c}{A}$ Reduced throughput

$$q = \frac{\rho . C}{\rho_c . C_c}\frac{\rho_0}{\rho_0} = \frac{\rho}{\rho_0}\frac{\rho_0}{\rho_c}\frac{C}{a_c} \tag{II.46}$$

$$q = \left(1 - \frac{\gamma-1}{\gamma+1}M^2\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}}\lambda \tag{II.47}$$

Given the connections between λ and M et $\frac{P}{P_0}$ et λ (or M)

This function can be calculated and tabulated according to the quantities.λ, M or $\frac{P}{P_0}$ For each gas, characterized by its value of γ.

This function reaches a maximum for $\lambda = 1, M = 1\ et\ \frac{P}{P_0} = \frac{P_c}{P_0}$ in critical conditions$q = 1$, by designate $\rho \quad . C \quad = \frac{\dot{m}}{A} = g_m$

g_m : Specific mass flow rate, it is observed that:

$q = \frac{g_m}{g_{m_c}} \Rightarrow g_m = q . g_{m_c}$

$$g_m = \underbrace{q \quad C_c}_{g_{m_c}} q \tag{II.48}$$

When :

$$q = 1 \Rightarrow g_{m_c} = \rho_c C_c = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \rho_0 \sqrt{2 \quad \frac{\gamma}{\gamma+1}\frac{P_0}{\rho_0}} \tag{II.49}$$

$$= \sqrt{\gamma \quad . P_0 . \rho_0 \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}} \left(\frac{2}{\gamma+1}\right)}$$

$$= \sqrt{\gamma \quad . P_0 . \rho_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$= g_{m_{max}} \quad ; \text{however } \rho_0 = \frac{P_0}{r \ \ T_0}$$

$$g_{m_{max}} = \beta(\gamma) \frac{P_0}{\sqrt{T_0}} \text{ or } \beta(\gamma) = \sqrt{\frac{\gamma}{r} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} \quad \text{(II.50)}$$

Therefore $\beta(\gamma)$ function of the nature of gas; $\gamma = 1.4, r = 287\, j/kg\ k, \beta(\gamma) = 0.04$

$$g_m = g_{m_{max}} q = \beta(\gamma) \frac{P_0}{\sqrt{T_0}}\ q$$

$$\dot{m} = g_m \ .A \quad = \beta(\gamma) \frac{P_0}{\sqrt{T_0}}\ q.A \quad \text{(II.51)}$$

same $A \quad = \frac{A_c}{q}$ and q go through a maximum for$\lambda = 1, M = 1$,and $\frac{P}{P_0} = \frac{P_c}{P_0}$, It can be concluded that critical conditions can only be achieved at the minimum cross-section of the pipeline.

Isentropic flow in a variable cross-section, types of nozzles.

Continuity equation:

$$\frac{d\rho}{\rho} + \frac{dC}{C} + \frac{dA}{A} = 0 \quad \text{(II.52)}$$

Equation de mouvement

$$\frac{d\rho}{\rho} = -M^2 \frac{dC}{C} \quad \text{(II.53)}$$

By eliminating dρ/ρ between the two equations, we will have:

$$-M^2 \frac{dC}{C} + \frac{dC}{C} + \frac{dA}{A} = 0 \quad \text{(II.54)}$$

$$\frac{dC}{C}(1 - M^2) + \frac{dA}{A} = 0$$

Hugoniot's formula

$$\frac{dA}{A} = \frac{dC}{C}(M^2 - 1) \quad \text{(II.55)}$$

The result obtained has been expressed in the form of three **Hugoniot theorems**:

A. In a subsonic flow region (M<1), the velocity and the area of the cross-section vary in opposite directions.

B. In a supersonic flow region (M>1), the velocity and the area of the cross-section vary in the same direction.

C. The velocity can only equal the speed of sound at a section of minimal air flow – the throat.

II.4 Study of a flow in a nozzle: convergent and convergent-divergent

Therefore, we can revert to the continuity equation to substitute dC/C with its value, resulting in.

$$\frac{d\rho}{\rho}+\left(\frac{1}{M^2-1}+1\right)\frac{dA}{A}=0 \quad \text{where} \quad \frac{d\rho}{\rho}=\left(\frac{M^2}{1-M^2}\right)\frac{dA}{A} \qquad \text{(II.56)}$$

Knowing that for the isentropic transformation.

$$\boxed{\frac{dP}{P}=\left(\frac{\gamma M^2}{1-M^2}\right)\frac{dA}{A}} \qquad \text{(II.57)}$$

The derived formulas enable the analysis of the variations in flow parameters as a function of dA, for M<1 and M>1.

Additionally, the list can be supplemented with formulas for the variations in temperature and sound speed.

State equation:

$$\frac{dP}{P}=\frac{d\rho}{\rho}+\frac{dT}{T}\Rightarrow\frac{dT}{T}=\left(\frac{\gamma M^2}{1-M^2}-\frac{M^2}{1-M^2}\right)\frac{dA}{A} \qquad \text{(II.58)}$$

$$\frac{dT}{T}=\frac{(\gamma-1\ )M^2}{(1-M^2)}\frac{dA}{A}$$

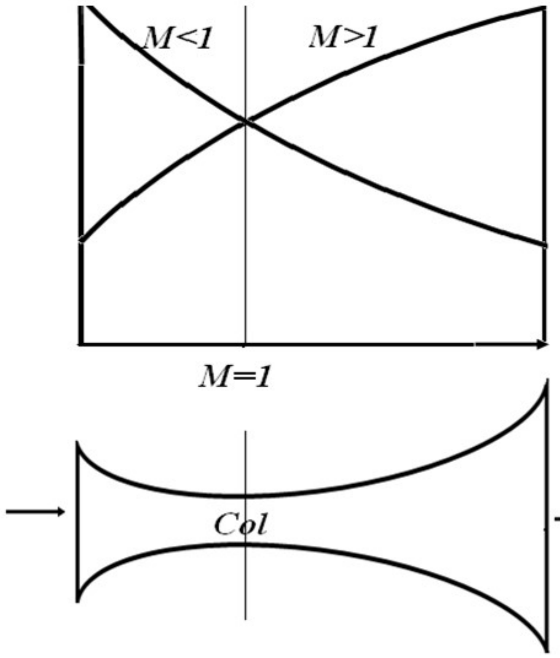
Velocity of sound:

$$a=\sqrt{\gamma\, rT}\ =\frac{da}{a}=\frac{1}{2}\frac{dT}{T} \qquad \text{(II.59)}$$

Ultimately :

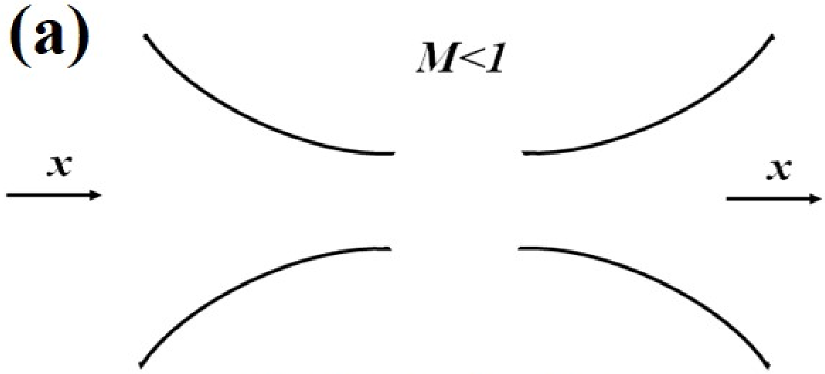
$$\frac{da}{a}=\frac{\gamma-1}{2}\ \frac{M^2}{(1-M^2)}\frac{dA}{A} \qquad \text{(II.60)}$$

The results obtained allow for the following graphs to be plotted.

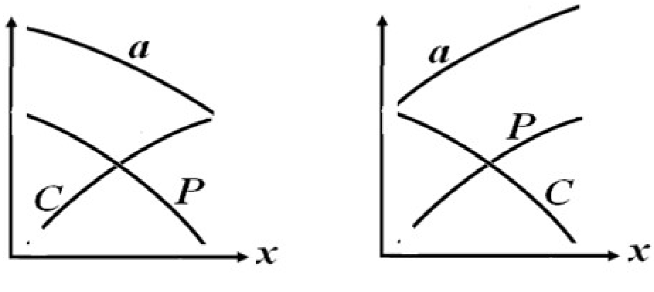


Section of ejection

(a)



Subsonic flow



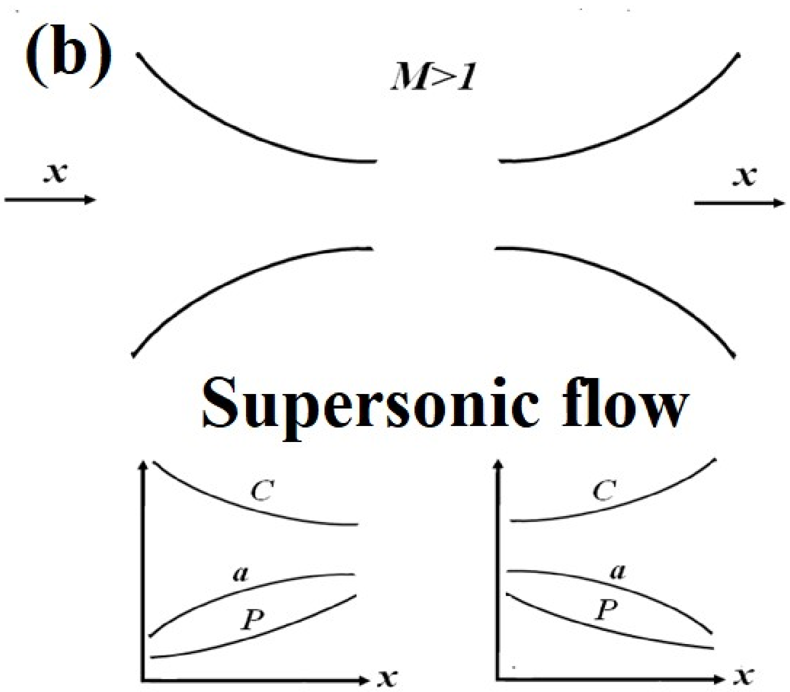
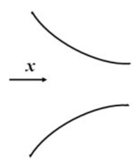
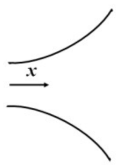
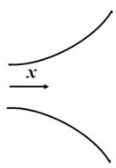
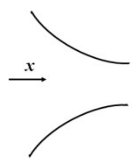


Figure II.1 a) subsonic and (b) supersonic flow regime in converging-diverging canalisations

In the technical field, it is customary to refer to the pipelines where dP>0 and dC<0 as 'Diffusers', while the nozzles where dP<0 and dC>0 are termed 'Tuyères'. Following this convention, the following diagram can be provided:

Nozzle	Flow regime	
	Subsonique	Supersonique
Nozzle		
Diffuser		

According to this diagram, it is clear that to produce a supersonic flow, a 'convergent-divergent' nozzle is required. This nozzle was proposed by 'Laval' in 1880. To ensure the proper functioning of the 'Laval' nozzle, a well-defined pressure must be applied at the exit section, which guarantees isentropic flow with critical speed at the throat and M > 1 in the exit section of the nozzle.

To transition from subsonic to supersonic, the nozzle must have a minimum section known as the 'throat'. In cases where the nozzle is 'choked', meaning that the flow is supersonic at a certain point, the conditions at the throat are considered critical conditions. In this scenario, the flow rate of the nozzle no longer depends on the downstream conditions.

- This phenomenon is referred to as the 'choking phenomenon'.

The 'choked' flow rate is given by:

$$\dot{m} = g_m \ . A \quad = \beta(\gamma) \frac{P_0}{\sqrt{T_0}} \ q . A \tag{II.61}$$

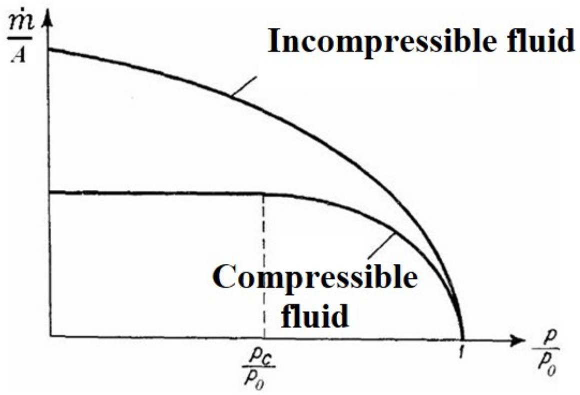


Figure II. 2 Variation of flow rate with P/P_0

It is observed that for the compressible fluid:

if $P/P_0 > P_c/P_0$ The flow rate ranges from 0 to the maximum flow rate**.**

if $P/P_0 < P_c/P_0$,

Such a regime is referred to as Adapt"é". Conversely, if M<1 at the nozzle exit, the regime is termed “**disadapted**”, and shock waves form within the nozzle.

Practical calculation of nozzle profiles

According to the continuity.

$$\rho_1 . A_1 . C_1 = \rho_2 . A_2 . C_2 \tag{II.62}$$

$$\frac{A_1}{A_2} = \frac{\rho_2}{\rho_1} . \frac{C_2}{C_1}$$

AN : Attention, never use volumetric flow rate in compressible fluid mechanics.

For an isentropic flow.

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1} \tag{II.63}$$

$$\frac{C_2}{C_1} = \frac{M_2}{M_1}.\frac{a_2}{a_1} = \frac{M_2}{M_1}.\frac{\sqrt{\gamma.r.T_2}}{\sqrt{\gamma.r.T_1}}$$

$$= \frac{M_2}{M_1}.\sqrt{\frac{T_2}{T_1}}$$

From where:

$$\frac{A_1}{A_2} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}.\frac{M_2}{M_1}.\sqrt{\frac{T_2}{T_1}}$$

$$\frac{A_1}{A_2} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1}.\frac{M_2}{M_1}.\left(\frac{T_2}{T_1}\right)^{1/2}$$

$$\frac{A_1}{A_2} = \left(\frac{T_2}{T_1}\right)^{\gamma+1/2(\gamma-1)}.\frac{M_2}{M_1} \tag{II.64}$$

Calcul$\frac{T_2}{T_1}$= ?

$$\begin{cases} \frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \\ \frac{T_0}{T_2} = 1 + \frac{\gamma-1}{2}M_2^2 \end{cases} \Rightarrow \frac{T_0}{T_1}/\frac{T_0}{T_2} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} \tag{II.65}$$

$$\Rightarrow \frac{T_0}{T_1}.\frac{T_2}{T_0} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} \tag{II.66}$$

For an isentropic flow, we also have:

$$\frac{P_2}{P_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma/\gamma-1} \tag{II.67}$$

$$\frac{\rho_2}{\rho_1} = \left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/\gamma-1} \tag{II.68}$$

Therefore :

$$\frac{A_1}{A_2} = \frac{M_2}{M_1}.\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma+1/2(\gamma-1)} \tag{II.69}$$

Generally, the critical section of the nozzle A_c is chosen as the reference.

Let us consider: $A_1 = A$, $M_1 = M$, $A_2 = A_c$, $M_2 = M_c = 1$

The previous results become:

$$\Rightarrow \frac{T_c}{T} = \frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right) \tag{II.70}$$

$$\frac{P_c}{P} = \left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{\gamma/\gamma-1} \tag{II.71}$$

$$\frac{\rho_c}{\rho} = \left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right)^{1/\gamma-1} \tag{II.72}$$

$$\frac{A}{A_c} = \frac{1}{M}.\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\gamma+1/2(\gamma-1)} \tag{II.73}$$

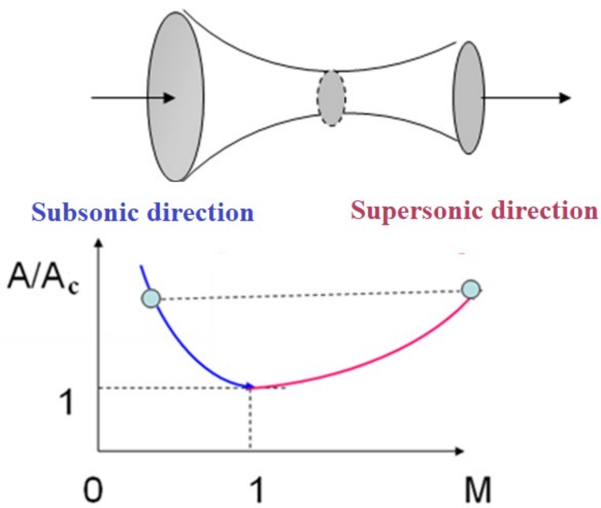


Figure II. 3 Variation of the A/A_c report as a function of Mach downstream

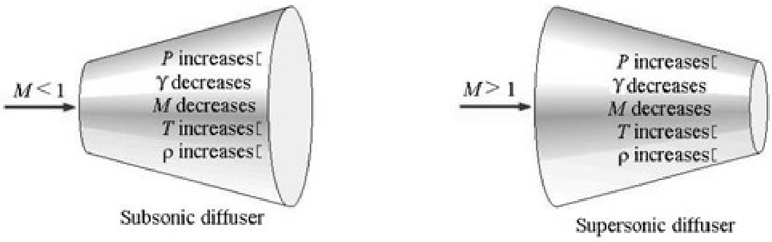
$$M << 1 : \frac{A}{A_c} \to \frac{1}{M}$$

$$M >> 1 : \frac{A}{A_c} \to \left(\frac{\gamma-1}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} M^{\frac{2}{\gamma-1}}$$

II.5 Overview of Subsonic and Supersonic Diffuser

Main characteristics of diffuser:

A. Incoming flow can be either subsonic or supersonic based on the application.
B. A diffuser is designed such that the loss in total pressure is minimal during the slowing down of the flow.
C. Diffusers are commonly used in propulsion systems such as air breathing engines, rocket engines etc
D. Diffusers are also integral components in many wind tunnel designs.



(a) Subsonic flow (b) Supersonic flow

CHAPTER III: SHOCK WAVES

When a fixed obstacle is placed in a supersonic gas flow, the non-isentropic stagnation of the fluid is marked by the emergence of a shock wave.

Part 1: Normal shock wave

Under certain conditions, very thin and irreversible discontinuities may occur in isentropic flows. These discontinuities are referred to as shock waves, and they are termed normal shock waves when they are perpendicular to the flow velocity vectors.

A normal shock wave in a one-dimensional flow is illustrated in the following figure:

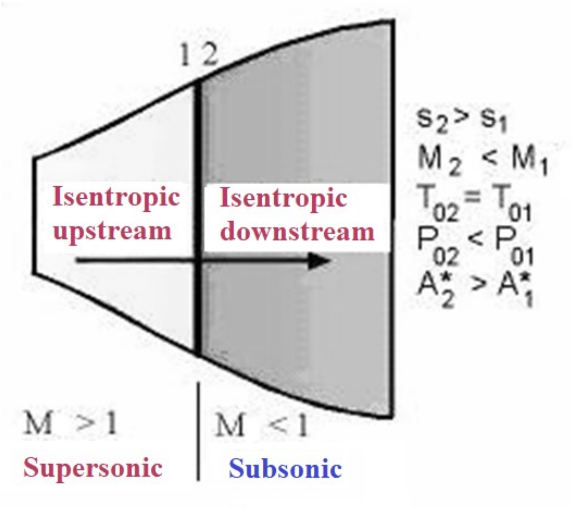


Figure III. 1Normal shock wave

III.2 Fundamental equations (continuity, momentum, energy) and Prandtl's relation

Let us characterize the conditions before and after the shock wave as 1 and 2.

A. Preservation of mass flow rate

The thickness of the shock wave is on the order of a few micrometers at most. Therefore, we have.

$$\rho_1 C_1 = \rho_2 C_2 \quad \text{(III.1)}$$

B. **Euler's Theorem:** By applying it to the material system formed by the wave, one obtains:

$$q_m(C_2 - C_1) = A \quad (P_1 - P_2) \quad \text{(III.2)}$$

Considering the

$$q_m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2 \quad \text{(III.3)}$$

We express this in the form of:

$$P_1 + \rho_1 C_1^2 = P_2 + \rho_2 C_2^2 \quad \text{(III.4)}$$

The sum $P \quad + \rho \quad C^2$ is referred to as Dynalpie of the fluid.

C. **Barre-Saint Venant Theorem:** All forms of this theorem applicable to irreversible adiabatic evolutions are valid:

$c_pT_1 + \frac{C_1^2}{2} = c_pT_2 + \frac{C_2^2}{2} = c_pT_0$ (Temperature Stagnation Conservation) (III.5)

$$\frac{\gamma}{\gamma-1}\frac{P_1}{\rho_1}+\frac{C_1^2}{2}=\frac{\gamma}{\gamma-1}\frac{P_2}{\rho_2}+\frac{C_2^2}{2}$$

$$\frac{T_0}{T}=1+\frac{\gamma-1}{2}M^2 \quad \text{(III.6)}$$

D. Characteristic equation of the fluid

$$\frac{P}{\rho}=r\,T \quad \text{(III.7)}$$

Relationships between the characteristic reports of the straight wave $\frac{P_2}{P_1},\frac{\rho_2}{\rho_1},\frac{T_2}{T_2}$

The Euler theorem can be expressed :

$$\rho_1\,A\,C_1(C_2-C_1)=\;A\;\;(P_1-P_2) \quad \text{(III.8)}$$

We multiply both members by C_1+C_2 to make appear $C_2^2-C_1^2$,

$$\rho_1\;\;C_1(C_2^2-C_1^2)=\;(P_1-P_2)(C_1+C_2) \quad \text{(III.9)}$$

We divide both members by $\rho_1C_1=\rho_2C_2$:

$$C_2^2-C_1^2=(P_1-P_2)\left(\frac{C_1}{\rho_1C_1}+\frac{C_2}{\rho_2C_2}\right)=(P_1-P_2)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right) \quad \text{(III.10)}$$

By incorporating this expression into the Barré-Saint Venant theorem:

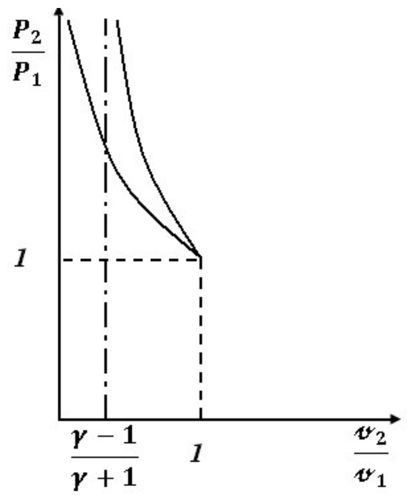
$$\frac{C_2^2-C_1^2}{2}=\frac{\gamma}{\gamma-1}\left(\frac{P_1}{\rho_1}-\frac{P_2}{\rho_2}\right)=\frac{1}{2}(P_1-P_2)\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=h_1-h_2 \quad \text{(III.11)}$$

This equation is referred to as the Rankine-Hugoniot equation for shock waves, or alternatively, as the Adiabatic-Dynamic equation. It can be expressed in the form:

$$\frac{P_2}{P_1}=\frac{1-\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1}}{\frac{\rho_2}{\rho_1}-\frac{\gamma+1}{\gamma-1}} \quad \text{(III.12)}$$

$$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma + 1}{\gamma - 1}\frac{P_2}{P_1}}{\frac{P_2}{P_1} + \frac{\gamma + 1}{\gamma - 1}} = \frac{v_1}{v_2} \quad \text{(III.13)}$$

One will notice the difference with the isentropic law. $\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{v_1}{v_2}\right)^\gamma$



By comparing the curves $\frac{P_2}{P_1} = f\left(\frac{v_2}{v_1}\right)$

The figure shown above is obtained. In the

The isentropic asymptote is the vertical axis.

In straight wave compression, the asymptote is:

$$\frac{v_2}{v_1} = \frac{\gamma - 1}{\gamma + 1}$$

The volume v2 remains finite.

Fig III 2: $\frac{T_2}{T_1} = f(\frac{v_2}{v_1})$

The report P_2/P_1 believe faster than an isentropic compression of the same ratio v_2/v_1, ce which suggests a more rapid increase in temperature.

Indeed:

$$\frac{T_2}{T_1} = \frac{\rho_1}{\rho_2} \times \frac{P_2}{P_1} \quad \text{(III.14)}$$

Instead of:

$$\frac{T_2}{T_1} = \frac{\frac{P_2}{P_1} + \frac{\gamma + 1}{\gamma - 1}}{\frac{P_1}{P_2} + \frac{\gamma + 1}{\gamma - 1}}$$

From where

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} \quad \text{(III.15)}$$

In isentropic compression.

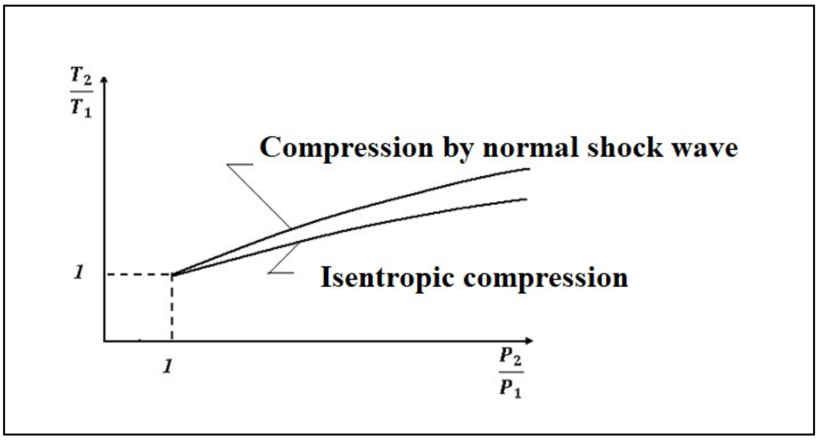


Fig III. 3: Comparaison $\frac{T_2}{T_1} = f\left(\frac{P_2}{P_1}\right)$ with and without normal shock wave

The figure illustrates the curves$\frac{T_2}{T_1} = f\left(\frac{P_2}{P_1}\right)$ In both cases.

A.5 Prandtl Relation

The Euler equation can be expressed:

$$C_2 - C_1 = \frac{P_1}{\rho_1 C_1} - \frac{P_2}{\rho_2 C_2} \tag{III.16}$$

Considering the expression for the speed of sound: $a^2 = \gamma P/\rho$

$$\gamma(C_2 - C_1) = \frac{a_1^2}{C_1} - \frac{a_2^2}{C_2} \tag{III.17}$$

The Barré-Saint Venant theorem becomes:

$$a_1^2 + \frac{\gamma - 1}{2} C_1^2 = a_2^2 + \frac{\gamma - 1}{2} C_2^2 = \frac{\gamma + 1}{2} a_c^2 \tag{III.18}$$

From where
$$a_1^2 = a_2^2 + (C_2^2 - C_1^2)\frac{\gamma - 1}{2}$$

The Euler equation is then expressed as:

$$a_1^2 = a_2^2 + (C_2^2 - C_1^2)\frac{\gamma - 1}{2}$$

$$\gamma(C_2 - C_1) = \frac{C_2^2 - C_1^2}{C_1}\frac{\gamma - 1}{2} + \frac{a_2^2}{C_1} - \frac{a_2^2}{C_2} \tag{III.19}$$

As a result: $$C_1\,C_2 = \frac{2}{\gamma+1}\left(a_2^2 + \frac{\gamma-1}{2}C_2^2\right)$$

And, by introducing the critical speed:

$$\boldsymbol{C_1\,C_2 = a_c^2}$$

III.2 Relationships of the normal shock wave in relation to the Mach number

III.2.1 Calculation of the characteristic shock wave ratios based on the upstream Mach number

The conservation of flow rate and the Prandtl relationship can be expressed in the form:

$$\frac{\rho_2}{\rho_1} = \frac{C_1}{C_2} = \frac{C_1^2}{a_c^2} \tag{III.20}$$

The Barré-Saint Venant theorem allows for the expression of a_c in relation to the upstream conditions.

$$a_c^2 = \frac{2}{\gamma+1}\left(a_1^2 + \frac{\gamma-1}{2}C_1^2\right) \tag{III.21}$$

From where does it derive:

$$\frac{\rho_2}{\rho_1} = \frac{C_1}{C_2} = \frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \tag{III.22}$$

$\frac{\rho_2}{\rho_1} = \frac{6M_1^2}{5+M_1^2}$ to provide $\gamma = 1.4$

By substituting $\frac{\rho_2}{\rho_1}$ by its value in the expression of $\frac{P_2}{P_1}$ view in (III.12) in summary, it can be stated as follows:

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1} \tag{III.23}$$

$$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \tag{III.24}$$

$\frac{P_2}{P_1} = \frac{7M_1^2-1}{6}$ to provide $\gamma = 1.4$

One can also express it in the form of :

$$M_1^2 = 1 + \frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right) \quad \text{(III.25)}$$

This indicates that a report $\frac{P_2}{P_1}$ greater than 1 can only be achieved if $M_1 > 1$.

It is noteworthy that, for $M_1 = 2$, $\frac{P_2}{P_1} = 4{,}5$, This indicates the potential for significant compression due to shock at the entrance of the propulsion reactors.

The ratio of absolute temperatures is provided by:

$$\frac{T_2}{T_1} = \frac{\rho_1}{\rho_2} \times \frac{P_2}{P_1} \quad \text{(III.26)}$$

From where

$$\frac{T_2}{T_1} = \frac{2\left(1 + \frac{\gamma-1}{2}M_1^2\right)[2\gamma M_1^2 - (\gamma-1)]}{(\gamma+1)^2 M_1^2} = \frac{(2 + (\gamma-1)M_1^2)[2\gamma M_1^2 - (\gamma-1)]}{(\gamma+1)^2 M_1^2} \quad \text{(III.23)}$$

$\frac{T_2}{T_1} = \frac{(M_1^2+5)(7M_1^2-1)}{36M_1^2}$ For $\gamma = 1.4$

III.2.2 Calculation of characteristic ratios based on upstream and downstream Mach numbers

Through a study similar to that of the previous paragraph, the relationship is established

The values of question

$$\frac{\rho_1}{\rho_2} = \frac{C_2}{C_1} = \frac{C_2^2}{a_c^2} = \frac{\frac{\gamma+1}{2}M_2^2}{1 + \frac{\gamma-1}{2}M_2^2} \quad \text{(IV.24)}$$

From where :

$$\frac{P_1}{P_2} = \frac{2\gamma}{\gamma+1}M_2^2 - \frac{\gamma-1}{\gamma+1} \quad \text{(IV.25)}$$

and

$$M_2^2 = 1 + \frac{\gamma+1}{2\gamma}\left(\frac{P_1}{P_2} - 1\right) \quad \text{(IV.26)}$$

It can be inferred that, as $P_1/P_2 < 1, M_2 < 1$.

In stating that, according to Prandtl's relationship, $\frac{C_1^2}{a_c^2} \times \frac{C_2^2}{a_c^2} = 1$

A relationship can be found between M_1 et M_2, which can be expressed in the form of:

$$M_2^2 = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma - 1}{2}} = \frac{2 + (\gamma - 1) M_1^2}{2\gamma M_1^2 - (\gamma - 1)} \tag{III.27}$$

For $\gamma = 1{,}4$

$$M_2^2 = \frac{5 + M_1^2}{7M_1^2 - 1} \tag{III.28}$$

This equation demonstrates that as $M_1 \; > 1$ and $\; M_2 \; < 1$.

The flow remains subsonic downstream of a straight shock wave.

Additionally, one can provide the relationship:

$$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2} \tag{III.29}$$

Let us finally recall the Barré-Saint Venant theorem and express it in the form:

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \tag{III.30}$$

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \tag{III.31}$$

$$\frac{T_2}{T_1} = \frac{2 + (\gamma - 1) M_1^2}{2 + (\gamma - 1) M_2^2} \tag{III.32}$$

III.2.3 Variation of entropy in a normal shock wave

The transformation in a Normal shock wave is irreversible and is accompanied by an increase in entropy.

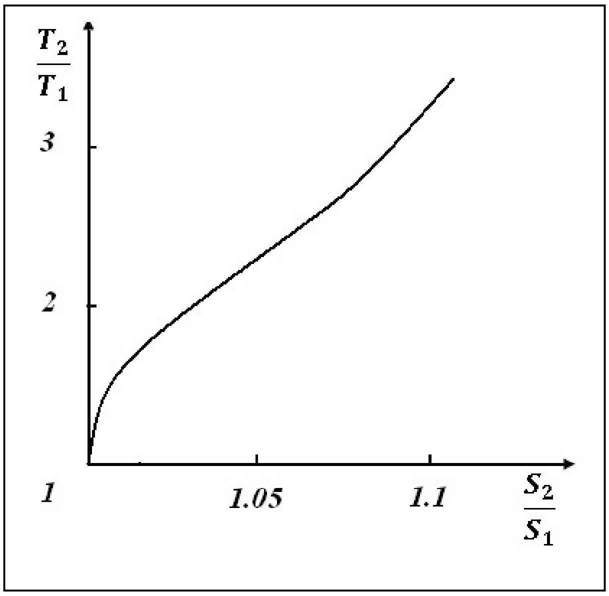


Fig IV. 4: Variation of $\frac{T_2}{T_1} = f\left(\frac{S_2}{S_1}\right)$ in a normal shock wave

One of the general expressions for the change in entropy of an ideal gas :

$$S_2 - S_1 = c_v ln \frac{P_2 v_2^{\gamma}}{P_1 v_1^{\gamma}} \quad \text{(IV.33)}$$

This expression allows for an immediate verification that $S_2 - S_1$ equals zero for an isentropic process. ($P v^{\gamma} = cte$).

Upon traversing a straight shock wave, one can express:

$$S_2 - S_1 = c_v ln \left[\frac{P_2}{P_1} \cdot \left(\frac{\rho_1}{\rho_2}\right)^{\gamma}\right] \quad \text{(IV.34)}$$

And replaceP_1/P_2 and ρ_1/ρ_2 through one of the previously established expressions.

$$S_2 - S_1 = c_v ln \left[\frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1} \cdot \left(\frac{1 + \frac{\gamma-1}{2} M_1^2}{\frac{\gamma+1}{2} M_1^2}\right)^{\gamma}\right] \quad \text{(IV.35)}$$

This variation of entropy is low for values of M_1 close to 1 (practically negligible up to M_1=1.2) and then increases rapidly.It should be noted that if M_1<1, this expression yields a negative change in entropy, which has no physical significance. Therefore, the right shock wave can only exist if M_1>1.The previous figure illustrates the change in entropy during the passage of a right wave on an entropy diagram where T_1 and S_1 have been selected as units. It can be demonstrated that for S_2=S_1, the curve has a third-order contact with the isentrope (vertical axis).

III.3 Limit cases: weak shock waves & strong shock waves

On a

$\frac{C_{2n}}{C_{1n}} = \frac{C_2^2}{a_c^2} = \frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}$, According to (III.22) and $C_{2t} = C_{1t} \cos\beta$, From which we deduce the components of C_2 by projecting along the axes *Ox* and *Oy*.

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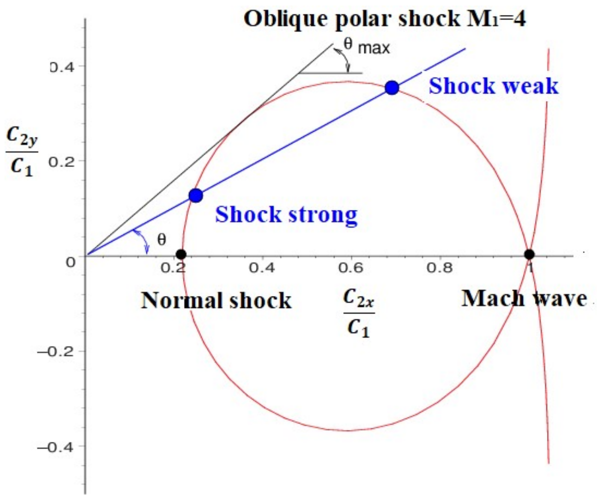


$\frac{C_{2x}}{C_1} = \boldsymbol{F(M1, \beta)}$ *and* $\frac{C_{2y}}{C_1} = \boldsymbol{G(M1, \beta)}$. For a fixed value of M_1, It can therefore be deduced for each value of β (angle of shock) the components of the speed downstream C_2, and consequently the angle of deflection $\theta = arctg(C_{2y}/C_{2x})$.

III.5 Pitot tube in supersonic conditions

A Pitot tube is a small hollow tube placed parallel to the flow direction in a supersonic wind tunnel to measure the flow Mach number by connecting it to a pressure transducer that measures stagnation pressure.

II.5.1 operating principle

The tube is linked to a pressure transducer that gauges the stagnation pressure located behind the bow shock. The free flow streamline, which is aligned with the tube axis, intersects the normal section of the bow shock. Consequently, the Rankine-Hugoniot relations applicable to a normal shock wave are relevant in this scenario. For instance, the equation below illustrates the relationship between the stagnation pressures upstream and downstream of the normal shock wave (P_{01} and P_{02}, respectively) as a function of the upstream flow Mach number, denoted as M_1. Therefore, by measuring P_{01} and P_{02}, one can determine the upstream flow Mach number.

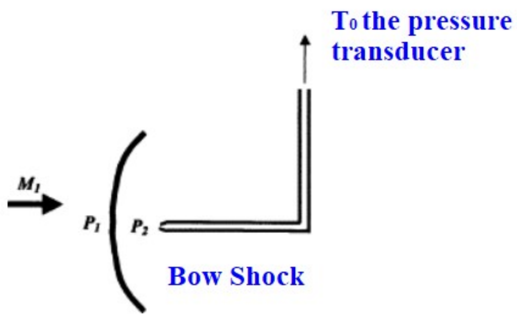


Figure III.5 Pitot probe

III.5.1 Calculation of the stagnation pressure following the Normal shock wave

After the Normal shock wave, the flow becomes isentropic once more, and the stagnation pressure, which is equal to the new generating pressure P_i', is provided by:

$$\frac{P_i'}{P} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/\gamma - 1} \quad \text{(IV.36)}$$

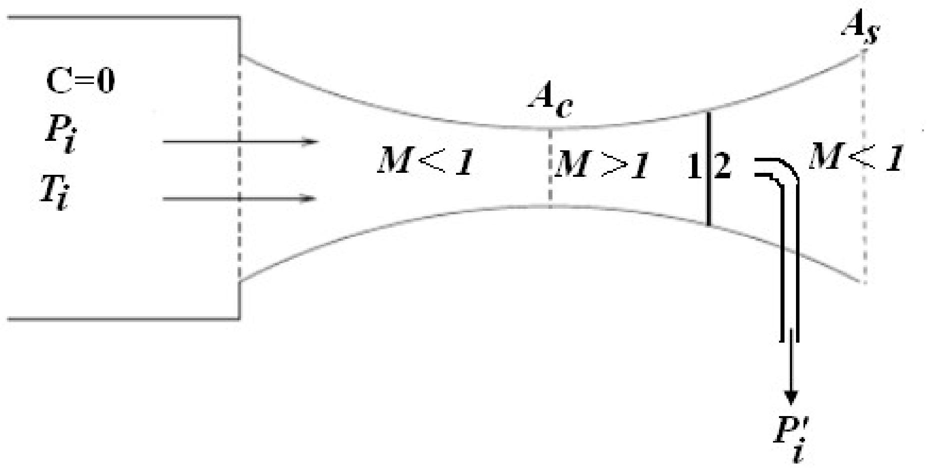


Figure III. 6 Variation of the stagnation pressure after the normal shock wave

The preceding calculations thus enable the determination of P_2 and subsequently Pr, where this pressure $Pr = P_i'$ is lower than the initial generating pressure of the flow, due to the irreversibility of the shock wave. The ratio $\frac{P_i'}{P_i}$ serves as a measure of irreversibility. of the shock wave.

The changes occurring upstream and downstream of the shock wave are isentropic, thus we have:

$$S_2 - S_1 = S_i' - S_i = r\, ln\frac{P_i}{P_i'} + c_p ln\frac{T_i'}{T_i} \quad \text{(IV.37)}$$

Since the stagnation temperature is maintained, even during an irreversible adiabatic process. ($T_i' = T_i \quad c.a\, d\, T_{02} = T_{01}$), on a :

$$S_2 - S_1 = r\, ln\frac{P_i}{P_i'} \quad \text{(IV.38)}$$

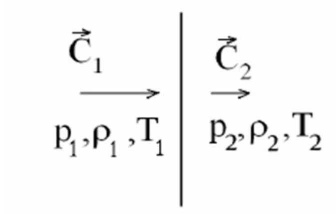
In comparison with the expression of $S_2 - S_1$ vue in equation (IV.35), one obtains:

$$\frac{P_i}{P_i'} = \left[\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\cdot\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{\frac{\gamma+1}{2}M_1^2}\right)^{\gamma}\right]^{1/\gamma-1} \quad \text{(IV.39)}$$

$$\boxed{\frac{P_i}{P_i'} = \left[\frac{2\gamma\ M_1^2-(\gamma-1)}{\gamma+1}\cdot\left(\frac{2+(\gamma-1)M_1^2}{(\gamma+1)^2M_1^2}\right)^{\gamma}\right]^{1/\gamma-1}} \quad \text{(IV.40)}$$

III.6 Variation of the section in a normal shock wave

By combining the mass flow rate equation $q_m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$ with isentropic relations



Normal stationary shock wave

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{1/\gamma-1} \quad \text{et}$$

$$\frac{C_2}{C_1} = \frac{M_2}{M_1}\cdot\frac{a_2}{a_1} = \frac{M_2}{M_1}\cdot\frac{\sqrt{\gamma.r.T_2}}{\sqrt{\gamma.r.T_1}}$$

$$\frac{A_2}{A_1} = \frac{M_2}{M_1}\cdot\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{\gamma+1/2(\gamma-1)} \quad \text{(IV.41)}$$

Therefore, in conclusion:

$$\frac{A_2}{A_1} = \frac{M_2}{M_1}\cdot\left(\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right)^{(\gamma+1)/2(\gamma-1)} \quad \text{(IV.42)}$$

III.7 Isentropic efficiency of compression by shock wave

To define this yield, we compare the Normal shock wave and the isentropic compression at the same pressures.

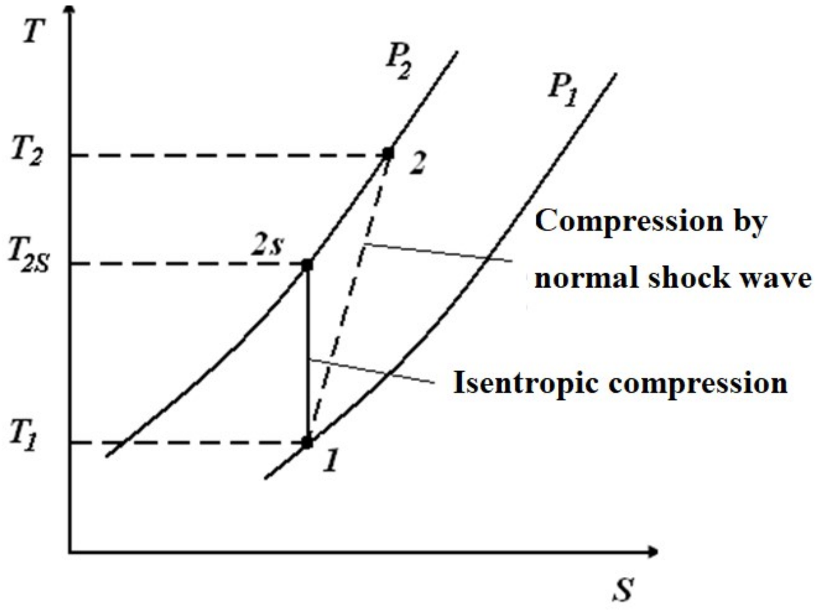


Figure III. 6 Isentropic and real détente in a nozzle

By analogy with the standard definitions used for turbochargers, the isentropic efficiency is defined as the ratio of the change in enthalpy during isentropic compression to the change in enthalpy during shock wave compression.

Let us denote points 1, 2s, and 2 as the corresponding figurative points (figure IV.6). :

$$\eta_s = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} = \frac{\frac{T_{2s}}{T_1} - 1}{\frac{T_2}{T_1} - 1} \quad \text{(IV.43)}$$

By substituting the equations from the previous paragraph into these equations, the following curves are obtained.

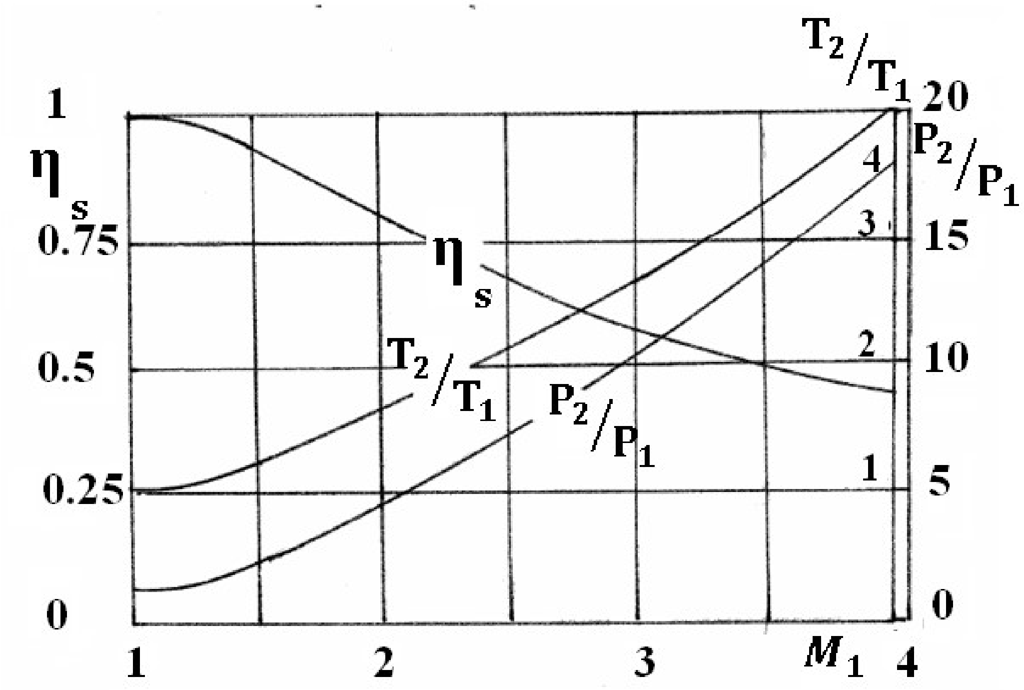


Figure III. 7 Summary graphs of the flow parameters before and after the shock wave

The efficiency of the straight wave surpasses that of the best machines for $P_2/P_1 \leq 2{,}7(M \approx 1.5\,, \eta \approx 0.9)$ It remains acceptable until $P_2/P_1 = 4.5\ \ (M \approx 2, \eta_s \approx 0.8)$.

IV.8 Examples of flows with shock

- At the conclusion of the previous paragraph, we observed the situation concerning improperly designed nozzles. A stationary normal shock wave is established at a location where the exit pressure equals the ambient pressure.
- External flows around the leading edge or the nose of a projectile, an aircraft, or a space shuttle... in supersonic conditions. Air intakes of the first supersonic aircraft.

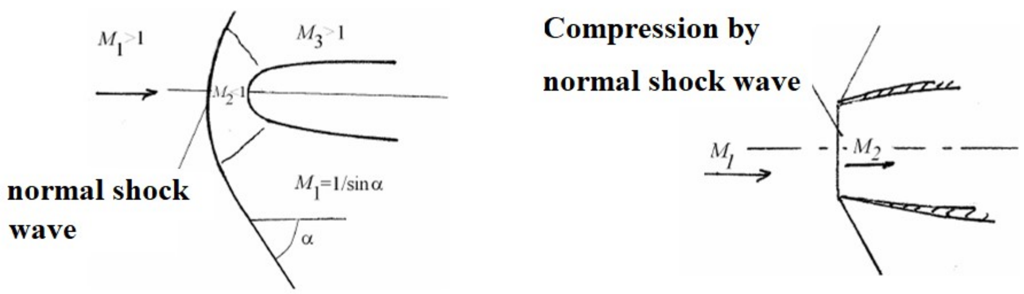


Figure III. 8 Example of flow with a normal shock wave

Comments:

- Beyond M=2, the isentropic efficiency becomes significantly low. We will see that it is preferable to use oblique shock waves.
- Very high compression ratios can be achieved through shock waves.
- Starting from M=3, the heating of the leading edges (T=1200°K) necessitates the use of refractory materials.
- The Normal shock wave is highly dissipative beyond ***M*=2**.
  - **<u>In summary:</u>**

  1. The shock represents a discontinuity.
  2. It reduces supersonic flow to subsonic flow.
  3. The thickness of the shock is minimal, measuring only a few micrometers.
  4. This is an irreversible phenomenon.

Energy conservation across the shock is isothermal: $T_{02} = T_{01}$.
The solution to these equations is frequently achieved through an iterative method. When utilizing these equations, it is crucial to pay particular attention to the upstream and downstream conditions of the shock:
1. The Mach number upstream is always supersonic, while the downstream Mach number is subsonic.
2. Stagnation pressures and densities decrease in the flow direction across the shock, whereas the stagnation temperature remains constant due to the adiabatic condition.
3. Pressure increases significantly, while temperature and density rise moderately downstream across the shock.
4. The area of the critical or sonic throat changes across the normal shock in the downstream direction, with A2 being greater than A_1.
5. Shock waves are highly irreversible, resulting in the specific entropy downstream (S_2) being greater than that upstream (S_1).
Moving normal shock waves, such as those occurring in explosions or spacecraft re-entering the atmosphere, can be analyzed as stationary normal shock waves by employing a coordinate system that moves with the wave speed in the direction of the shock wave.

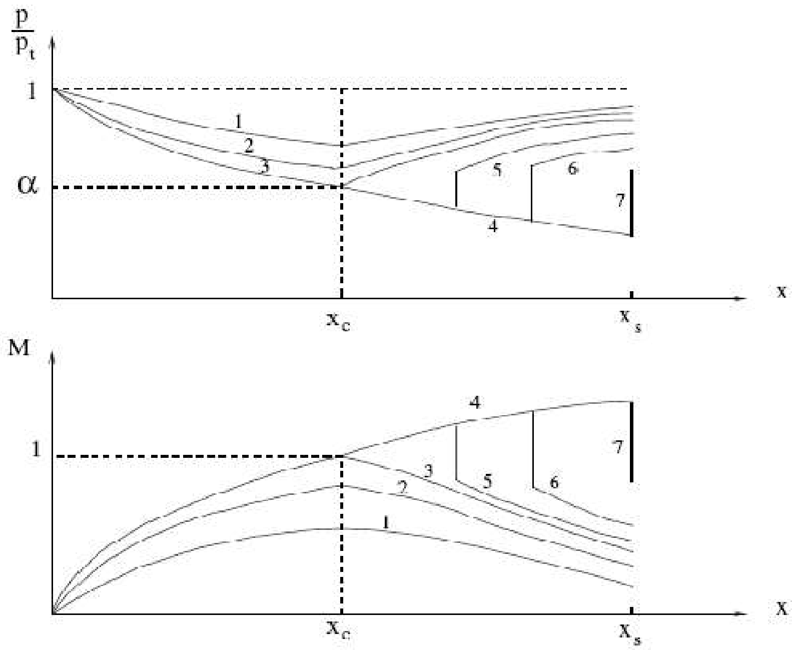


Figure III. 9: Flow regimes in a Laval nozzle

Part 2:OBLIQUE SHOCK WAVE

Let us consider a supersonic flow occurring along the surfaces of a dihedral angle (π-Ψ).

The direction of the velocity vector $(C_1)^{\rightarrow}$ is abruptly altered.

The downstream flow may be subsonic or supersonic depending on the magnitude of the deflection Ψ.

III.2. 1 Concepts of Oblique Shock Waves

Interprétation 1.1: Compression through oblique shock

a) Consider a supersonic flow along a dihedral angle causing a deviation Ψ, an oblique wave Ω was generated.

b) b) Preservation of mass flow rate: Let us consider a unit section AB on the wave Ω and the fluid particles that have passed through this section during the time interval dt. In the following figure, we have:

The elemental mass dm that has traversed AB during the time dt is

$$dm = \rho_1 C_{1n}\, dt = \rho_2 C_{2n}\, dt \qquad \text{(V.1)}$$

C_{1n} et C_{2n} being the components of the speed normal to the wave. The continuity equation is thus expressed as:

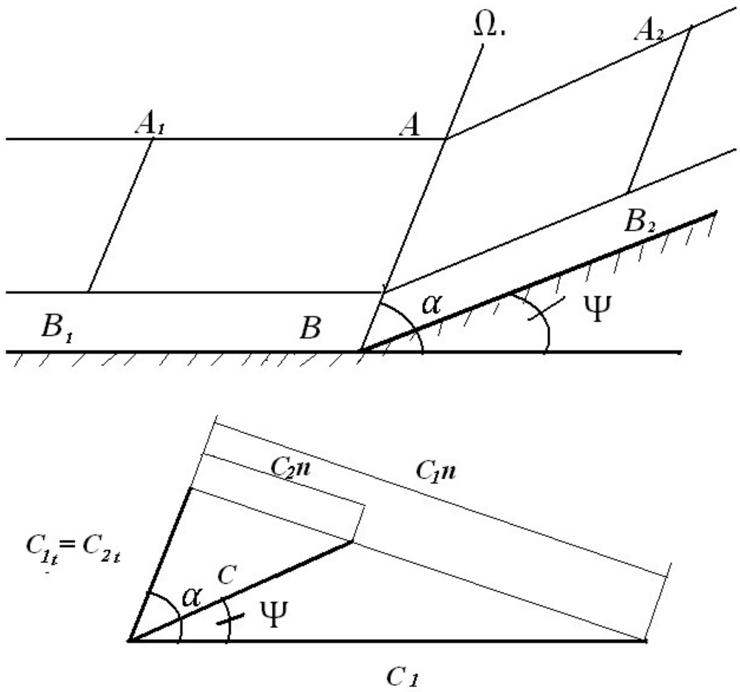


Figure III.6 Compression by oblique shock wave

III.2 Equations de base et relation de Prandtl

According to the figure III.6,

$$\boldsymbol{\rho_1 C_{1n} = \rho_2 C_{2n}} \quad \text{(V.2)}$$

c) Euler's Theorem: By applying it to the material system formed by the wave, one obtains:

$$q(C_{2t} - C_{1t}) = 0, \text{d'où} \quad C_{2t} = C_{1t} \quad \text{(V.3)}$$

It follows that the wave is perpendicular to the vector.$\overrightarrow{C_1} - \overrightarrow{C_2}$

Let us project onto the normal to the wave:

$$q(C_{2t} - C_{1t}) = S(P_1 - P_2) \quad \text{(V.4)}$$

For a unit section $q = \rho_1 C_{1n} = \rho_2 C_{2n}$, The Euler equation is thus expressed as :

$$\boldsymbol{P_1 + \rho_1 C_{1n}^2 = P_2 + \rho_2 C_{2n}^2} \quad \text{(V.5)}$$

d) Barré-Saint Venant Theorem: All forms of this theorem applicable to irreversible adiabatic processes can be utilized.

$$c_pT_1 + \frac{C_1^2}{2} = c_pT_2 + \frac{C_2^2}{2} = c_pT_i \quad \text{(V.6)}$$

By introducing the components of speed, we obtain:

$$c_pT_1 + \frac{C_{1n}^2}{2} + \frac{C_{1t}^2}{2} = c_pT_2 + \frac{C_{2n}^2}{2} + \frac{C_{2t}^2}{2} \quad \text{(V.7)}$$

Considering the equality of the tangential components

$$c_pT_1 + \frac{C_{1n}^2}{2} = c_pT_2 + \frac{C_{2n}^2}{2} \quad \text{(V.8)}$$

III.2.1 Conclusion

Key findings: By comparing the previous equations to the fundamental equations of the right shock wave, it is observed that the equations for the oblique wave can be derived by substituting the speeds C_1 and C_2 with their normal components C_{1n} and C_{2n} in the equations of the right wave**.**

Thus, by substituting in all the expressions obtained in the Normal shock wave,

$C_{1n} = C_1 sin\alpha$, C_2 par C_{2n}, $M_1 = C_1/a$ by :

$$M_{1n} = \frac{C_{1n}}{a} = M_1 sin\alpha \quad \text{(V.9)}$$

$M_1 sin\alpha$ is referred to as the normal Mach number, $M_2 = C_2/a$ by :

$$M_{2n} = \frac{C_{2n}}{a} = M_2 \sin(\alpha - \psi) \quad \text{(V.10)}$$

For instance, one obtains:

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} M_1^2 sin^2\alpha - \frac{\gamma-1}{\gamma+1} \quad \text{(V.11)}$$

Note 01:

The compression rate of the oblique shock wave is therefore lower than that of the straight shock wave.

$$S_2 - S_1 = c_v ln\left[\frac{2\gamma}{\gamma+1} M_1^2 sin^2\alpha - \frac{\gamma-1}{\gamma+1} . \left(\frac{1 + \frac{\gamma-1}{2} M_1^2 sin^2\alpha}{\frac{\gamma+1}{2} M_1^2 sin^2\alpha}\right)^{\gamma}\right] \quad \text{(V.12)}$$

Note 02:

The increase in entropy is also lower than that of a straight wave, indicating a reduction in irreversible internal losses.

We also establish the relationships:

$$C_{1n}\,C_{2n} = a_c^2 - \frac{\gamma-1}{\gamma+1}C_t^2 \quad \boxed{\textbf{(Relation of Prandtl)}} \qquad \text{(V.13)}$$

$$M_1^2 sin^2(\alpha-\psi) = \frac{1+\frac{\gamma-1}{2}M_1^2 sin^2\alpha}{\gamma M_1^2 sin^2\alpha - \frac{\gamma-1}{2}} \qquad \text{(V.14)}$$

$$tg\psi = 2cotg\alpha\,\frac{M_1^2 sin^2\alpha - 1}{M_1^2(\gamma+cos2\alpha)+2} \qquad \text{(V.15)}$$

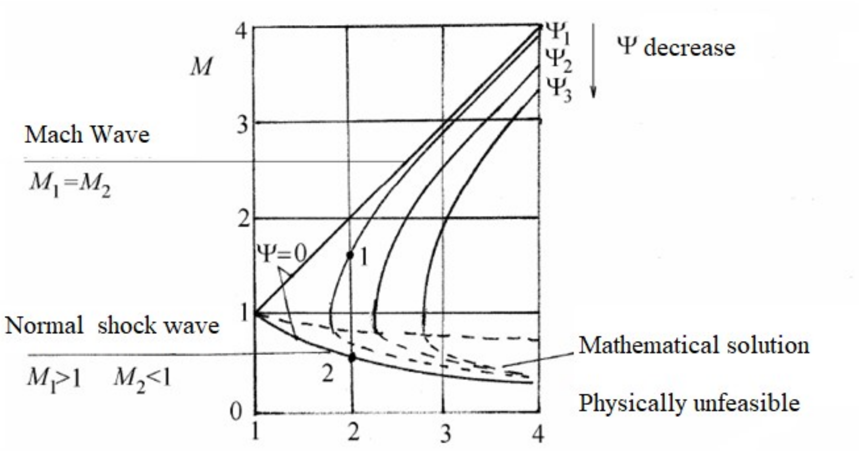


Figure III.7 Properties of the oblique wave

For a given value of M_1 and ψ, there are two values of M_2;

The oblique wave corresponds to the highest Mach number M_2, which is almost always supersonic, while the other solution lacks physical existence (discontinuous trait);

In the limit, for ψ=0, we find the two solutions α=90° (corresponding to the straight shock wave) and $\alpha = arc\,\sin\,(1/M_1)$, corresponding to the Mach wave, caused for instance by a slight surface defect on the wall of the flow, and for which $M_1 = M_2$.

III.2.2 Flow around a corner

Let us consider the flow around a salient dihedral angle at the apex.2ψ, The stagnation is normal at the vector.$\overrightarrow{C_1}$.

a) If ψ is small, the phenomenon can be inferred from the previous one by symmetry;

From a certain maximum value of ψ or a certain minimum value of M_1 that can be calculated, the wave detaches and the Mach number M_2 becomes less than 1.

b) If the obstacle is blunt, a similar phenomenon occurs, but the wave detaches earlier.

Relaxation through oblique waves

The previous calculations can be revisited with an angle ψ<0, resulting in a divergence of the flow. This leads to a decrease in entropy, thus rendering them devoid of physical significance .

Thus, we have

$$sin\alpha_1 = \frac{1}{M_1}, \; sin\alpha_2 = \frac{1}{M_2} \qquad \text{(V.16)}$$

Between these Mach waves, the change in direction occurs gradually. It is caused by a series of fine, fan-shaped oblique waves. The transformation is isentropic. As there is expansion $P_2 < P_1$, $M_2 > M_1$, $\alpha_2 < \alpha_1$

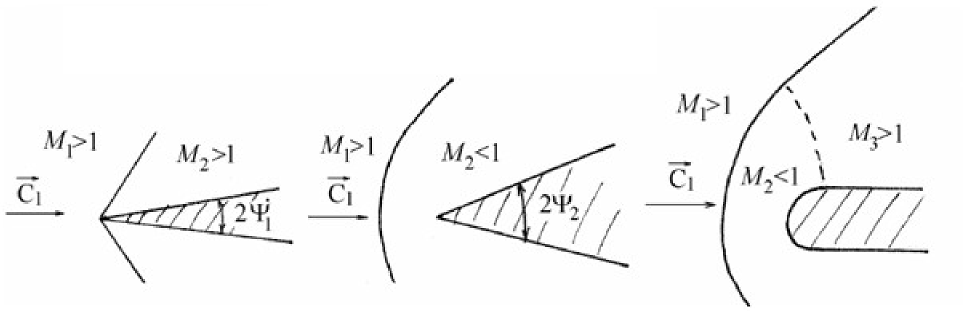


Figure III.8 several characteristic configurations where one can generally determine whether it is a strong or weak oblique shock.

III.2.3 Reflection and refraction of oblique waves

When an oblique shock wave encounters a wall, reflection occurs. The reflected shock generates a deflection in the opposite direction, with the downstream flow once again parallel to the wall.

Each wave results in a decrease in the Mach number and an increase in pressure. The compression rates of the two waves differ, as do the ratios of their Mach numbers. Figure (III.4) schematically illustrates the phenomenon along with the pressure variation along a streamline and at the wall.

A similar phenomenon occurs when there is an intersection of waves, as shown in figure (III.5), which is accompanied by refraction. This refraction is negligible for low-intensity waves.

When an oblique shock wave reflects off the free boundary of a jet, reflection occurs with a

Sign change.

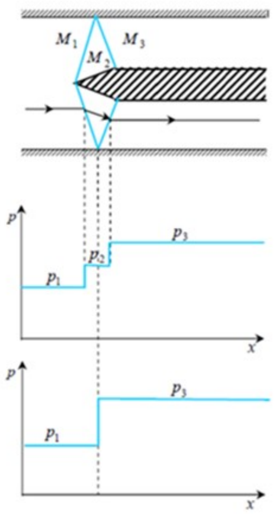


Figure III.9 Oblique shock wave reflection

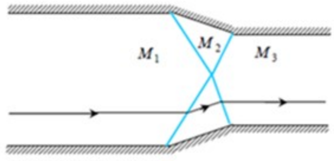


Figure III.10 Intersection of oblique shock waves

CHAPTER IV: Détente Prandtl-Mayer

IV.1Introduction

The Prandtl-Mayer expansion describes the isentropic expansion (without entropy loss) of a supersonic gas when it encounters a convex geometric discontinuity, such as an outward angle.

In contrast to an oblique shock (compression), the Prandtl-Mayer expansion involves a change in direction accompanied by a decrease in pressure, temperature, and density, while the speed of the gas increases.

IV.2 Prandtl-Mayer expansion wave

As an object traverses through a gas, the gas molecules are redirected around the object. If the object's speed is significantly lower than the speed of sound in the gas, the gas density remains unchanged, and the gas flow can be characterized by the conservation of momentum and energy. As the object's speed approaches the speed of sound, it becomes necessary to account for the compressibility effects on the gas. The gas density varies locally as it is compressed by the object. When an object exceeds the speed of sound, and there is a sudden reduction in the flow area, shock waves are produced. Conversely, if the flow area expands, a different flow phenomenon occurs. In cases of abrupt expansion, we encounter a centered expansion fan.

IV.3 Mechanism of expansion

The gas encounters a wall with a convex angle.

Since this is a supersonic flow, information cannot propagate upstream.

A range of weak Mach waves develops from the corner.

These waves create a fan-like structure known as the Prandtl-Mayer fan.

Each wave induces a slight deviation in the flow, resulting in a minor increase in speed and a decrease in pressure/temperature.

IV.4 Comparison of Shock Wave / Prandtl-Mayer Expansion

Aspect	Shock Wave	Expansion **Prandtl-Mayer**
Geometry	Entry Angle	Exit Angle
Processus	Non-isentropic (losses)	Isentropic (reversible)
Velocity	Decrease	Increase
Pressure/temperature	Increase	Decrease
Entropy	Increase	Constante

Applications

1. Supersonic nozzles (as used in rocket engines)
2. Aerodynamics of bodies with angular geometry
3. Supersonic wind tunnels
4. Studies of flow in divergent nozzles

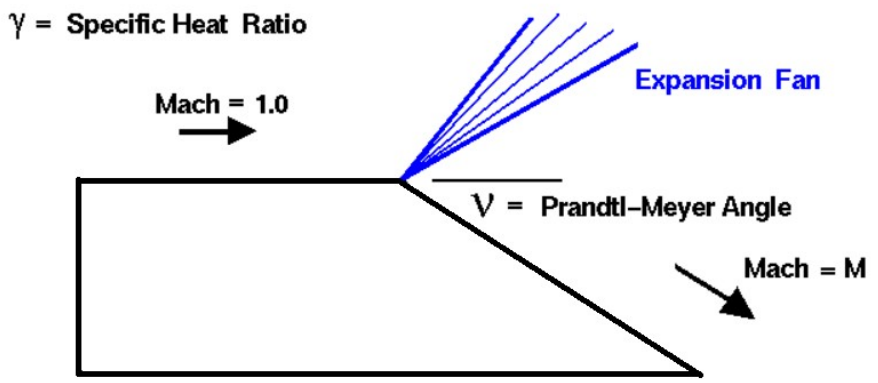


Figure IV.1 Prandtl-Mayer expansion wave

The calculation of the expansion fan necessitates the application of the Prandtl-Meyer function. This function is derived from the principles of conservation of mass, momentum, and energy for infinitesimal (differential) deflections. The Prandtl-Meyer function is represented by the Greek letter $\boldsymbol{\nu}$ in the presentation and is a function of the Mach number M and the specific heat ratio $\boldsymbol{\gamma}$ of the gas.

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - tan^{-1}\sqrt{(M^2-1)} \qquad \text{(IV.1)}$$

*v(M):*Prandtl-Mayer function (in radians)

M: Mach number

γ:Ratio of specific heats (γ=1.4 for air)

The change in angle θ corresponds to:

$$\theta = v(M_2 - M_1) \qquad \text{(IV.1)}$$

Where $M_1$and M_2 are the upstream and downstream Mach numbers of the expansion.

Properties of the Prandtl-Mayer fan

Quantity	Evolution in the range
Velocity	Increase
Pressure	Decrease
Temperature	Decrease
Density	Decrease
Mach Number	increase

Example

A supersonic airflow with M_1=2.0 approaches a 15° corner. Following the Prandtl-Mayer expansion fan, the flow deflects by 15° and the Mach number increases (for instance to M_1≈2.4), while the pressure, temperature, and density decrease.

CHAPTER V: Non-isentropic 1D flow in a constant cross-section conduit

Part 1: COMPRESSIBLE FLOW WITH FRICTION AND WITHOUT HEAT TRANSFER (FANNO'S THEORY)

V.1 Analysis of Fanno flow and fundamental equations

Chapter (III) demonstrated the impact of section change on compressible flow while disregarding heat transfer and friction. We can now incorporate friction and heat transfer into the section change and examine the coupled effects, which are discussed in advanced texts. However, as a basic introduction, this section only addresses the effect of friction, neglecting section change and heat transfer. The fundamental assumptions are:

- One-dimensional, steady-state, and adiabatic flow
- Ideal gas with constant specific heats
- Constant straight section conduit
- Negligible mechanical work and potential energy changes
- Wall shear stress correlated by a Darcy friction factor

Indeed, we are investigating a friction problem in Moody-type piping, but with significant variations in kinetic energy, enthalpy, and pressure within the fluid flow.

Let us consider the elementary control volume of the conduit with section A and length dx as depicted in Figure (V.1). The section remains constant, yet other flow properties (p, ρ, T, h, C) may vary with x.

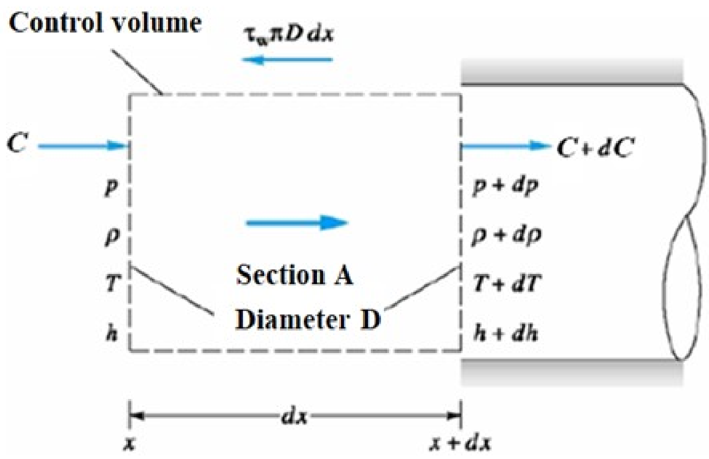


Figure V .1 Elementary control volume for a flow with friction in a conduit of constant cross-section

The application of the three conservation laws to this control volume results in three differential equations.

Continuity :

$$\rho \quad V \quad = \frac{\dot{m}}{A} = G = const \tag{V.1}$$

Where

$$\frac{d\rho}{\rho} + \frac{dV}{V} = 0 \tag{V.2}$$

Quantity of motion according to x:

$$P \quad A - (P + dP)A - \tau_w \pi D\, dx \quad = \dot{m}(C \quad + dC - C \quad) \tag{V.3}$$

Where

$$dP \quad + \frac{4\tau_w\, dx}{D} + \rho C\, dC = 0 \tag{V.4}$$

Energy :

$$h \quad + \frac{1}{2}C^2 \ = h_0 = C_P T_0 = C_P T \quad + \frac{1}{2}C^2 \tag{V.5}$$

Where

$$C_P dT \quad + C dC \quad = 0 \tag{V.6}$$

Since these three equations contain five unknowns: P, ρ, T, C, and τw, we require two complementary relationships. This is the ideal gas law.

$$P = \rho\, r\, T \quad ou \qquad \frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \tag{V.7}$$

To eliminate τ_w as an unknown, it is assumed that the shear stress at the wall is correlated by a local Darcy friction coefficient.λ

$$\tau_w = \frac{1}{8}\, \lambda\, \rho\, C^2 = \frac{1}{2} \lambda \gamma\, P\, M^2 \qquad \text{(V.8)}$$

Where the final form adheres to the speed relationship of sound in an ideal gas $a^2 = \gamma\, r\, T$. In practice, λ can be associated with the local Reynolds number and the roughness of the wall.

V.2 Variation of flow characteristics as a function of Mach number

The equations (V.4) and (V.7) are first-order differential equations that can be integrated using friction coefficient data, starting from any inlet section 1, where P_1, T_1, C_1, etc., are known, to determine P(x), T(x), etc., along the pipeline. It is virtually impossible to eliminate all but one variable to provide, for instance, a simple differential equation for P(x), but all equations can be expressed in terms of the Mach number M(x) and the friction coefficient, utilizing the definition of the Mach number.

$$C^2 = M^2\, \gamma\, rT \qquad \text{(V.8)}$$

Where

$$\frac{2dC}{C} = \frac{2dM}{M} + \frac{dT}{T} \qquad \text{(V.9)}$$

By eliminating variables between the equations (V.2 to V.9), we derive the working relationships.

$$dP \quad + \frac{4(\frac{1}{2}\lambda\gamma\, P\, M^2)dx}{D} + \rho C\, dC = 0$$

$$\frac{dP}{P} = -\gamma M^2 \frac{1+(\gamma-1)M^2}{2(1-M^2\,)} \lambda \frac{dx}{D} \qquad \text{(V.10)}$$

$$\frac{d\rho}{\rho} = \frac{\gamma\, M^2}{2(1-M^2\,)} \lambda \frac{dx}{D} = -\frac{dC}{C} \qquad \text{(V.11)}$$

$$\frac{dP_0}{P_0} = \frac{d\rho_0}{\rho_0} = -\frac{1}{2}\gamma M^2 \lambda \frac{dx}{D} \qquad \text{(V.12)}$$

$$\frac{dT}{T} = -\frac{\gamma(\gamma-1)M^4}{2(1-M^2\,)} \lambda \frac{dx}{D} \qquad \text{(V.13)}$$

$$\frac{dM^2}{M^2} = \gamma M^2 \frac{1+\frac{1}{2}(\gamma-1)M^2}{1-M^2} \lambda \frac{dx}{D} \qquad \text{(V.14)}$$

All except dP_0/P_0 have the factor $1\text{-}M^2$ in the denominator, so that, similar to the section change formulas in chapter (II), the subsonic and supersonic flows exhibit opposing effects:

Property	Subsonic	Supersonic
P	Decreases	Increases
ρ	Decreases	Increases
C	Increases	Decreases
P_0 ρ_0	Decreases	Decreases
T	Decreases	Increases
M	Increases	Decreases
Entropy	Increases	Increases

We have added to the list above that entropy must increase along the duct for the flow to be either subsonic or supersonic, as a consequence of the second law for adiabatic flow. For the same reason, both stagnation pressure and density must decrease.

The key parameter mentioned above is the Mach number. If the incoming flow is subsonic or supersonic, the Mach number of the duct always tends to decrease downstream. M =1, because it is the path along which entropy increases. If the pressure and density are The equations (V.10 to V.14) and the entropy from equation (V.35) have been calculated, and the result can be plotted in figure (V.2) as a function of the Mach number for γ = 1.4.

The maximum entropy occurs at M = 1, as the second law requires that the flow properties in the duct continuously approach the sonic point. SinceP_0 *and* ρ_0 continuously decrease along the duct due to friction losses (non-isentropic), they are not useful as reference properties. However, the sonic properties P^*, ρ^*, T^*, P_0^* *and* ρ_0^*, serve as the appropriate constant reference quantities in adiabatic flow within ducts. The theory then calculates the ratios P/P^* , T/T^* etc, as a function of the local Mach number and the integrated friction effect.

V.3 Coefficient of friction and entropy variation

To derive practical formulas, we first address equation (V.14), which connects the Mach number to friction. We will separate the variables and integrate:

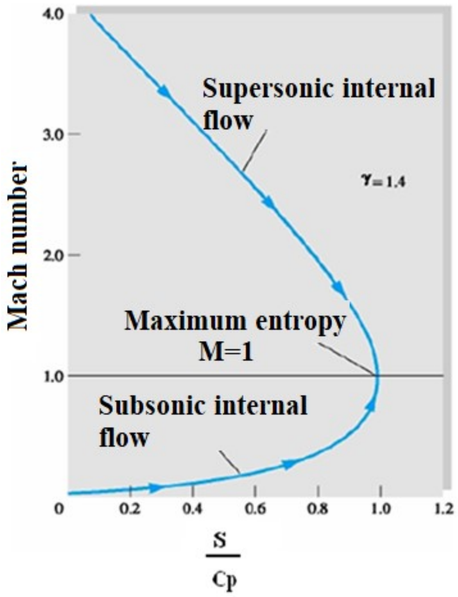
$$\int_0^{L^*} \lambda \frac{dx}{D} = \int_{M^2}^{1,0} \frac{1-M^2}{\gamma M^4 \left[1+\frac{1}{2}(\gamma-1\ \ )M^2\right]} dM^2 \tag{V.15}$$

The upper limit is the sonic point, whether it is actually reached or not in the flow of the conduit. The lower limit is arbitrarily set at the position x = 0, where the Mach number is M.

The result of the integration is

$$\frac{\bar{\lambda}L^*}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma} ln \frac{(\gamma+1\ \ )M^2}{2+(\gamma-1\ \ )M^2} \quad \text{(V.16)}$$

Where λ represents the average value of the friction coefficient between 0 and L^*. In practice, a mean λ is always assumed, and no effort is made to account for the slight variations in the Reynolds number along the pipe. For non-circular pipes, D is substituted with the hydraulic diameter. $D_h = (4 \times section)/périmètre$.



Adiabatic flow with friction in a constant cross-section duct always approaches M=1 to comply with the second law of thermodynamics.

The calculated curve is independent of the friction coefficient value.

The equation (V.17) is tabulated based on the Mach number. The length L* represents the required duct length to establish a flow in the duct transitioning from the Mach number M to the sonic point. Numerous issues involve short ducts that never reach sonic conditions, for which the solution utilizes the differences in the 'maximum' lengths, or sonic values, that are tabulated. For instance, the length ΔL required to develop from M_1 to M_2 is provided by

$$\bar{\lambda}\frac{\Delta L}{D} = \left(\frac{\bar{\lambda}L^*}{D}\right)_1 - \left(\frac{\bar{\lambda}L^*}{D}\right)_2 \quad \text{(V.17)}$$

This eliminates the necessity for separate tabs for short conduits.

It is advisable that the friction coefficient $\bar{\lambda}$ be estimated from the Moody diagram based on the average Reynolds number and the ratio of the conduit's wall roughness.

<u>Example 1:</u>

A subsonic airflow flows through an adiabatic duct with a diameter of 2cm. The average friction coefficient is 0.024. What is the required length of the duct to accelerate the fluid flow from M_1 = 0.1 to M_2 = 0.5? Additionally, what is the extra length needed to further accelerate it to M_3 = 1.0?

assume γ =1.4

Solution :

The equation (V.17) is applicable, using the values of $\frac{\bar{\lambda}L^*}{D}$ calculated from equation (V.16)

$$\bar{\lambda}\frac{\Delta L}{D} = \frac{0.024\Delta L}{0.02} = \left(\frac{\overline{\lambda L^*}}{D}\right)_{M=0.1} - \left(\frac{\overline{\lambda L^*}}{D}\right)_{M=0.5}$$

=66.9216-1.0691=65.8525

Thus $\Delta L = \frac{65.8525}{0.024} = 55m$

These calculations are typical: It takes 55 meters to accelerate to M = 0.5 and then only an additional 0.9 meters to fully reach the sonic point.

The formulas for other flow properties along the conduit can be derived from equations (V.I0 to V.14). Equation (V.15) can be used to eliminate $\lambda\, dx/D$.

The complementary length $\Delta L'$required to transition from M = 0.5 to M = 1.0 is taken directly from table (II).

$$\lambda\frac{\Delta L'}{D} = \left(\frac{\lambda L^*}{D}\right)_{M=0.5} = 1.0691$$

$$\Delta L' = L^*{}_{M=0.5} = \frac{1.0691(0.02)}{0.024} = 0.9m$$

These calculations are typical: It takes 55m to accelerate to M = 0.5 and then only an additional 0.9 m to fully reach the sonic point.

The formulas for other flow properties along the conduit can be derived from equations (V.10 to V.14). Equation (V.15) can be used to eliminate λ dx/D from each of the other relations, given, for instance, dP/P as a function solely of M et dM^2/M^2. For convenience in tabulating the results, each expression is then fully integrated from *(P, M)* to the sonic point *(P*, 1.0)*. The integrated results are:

$$\frac{P}{P^*} = \frac{1}{M}\left[\frac{\gamma+1}{2+(\gamma-1)M^2}\right]^{1/2}$$

$$\frac{\rho}{\rho^*} = \frac{C}{C^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{1/2}$$

$$\frac{T}{T^*} = \frac{a^2}{a^{*2}} = \frac{\gamma+1}{2+(\gamma-1)M^2}$$

$$\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{(1/2)\,(\gamma+1)/(\gamma-1)}$$

All of these proportions are also tabulated. To calculate the changes between points M_1 and M_2 that are not sonic, the products of these proportions are utilized. For instance,

$$\frac{P_2}{P_1} = \frac{P_2}{P^*}\frac{P^*}{P_1}$$

Since P^* is a constant reference value for the flow.

Example2:

For the internal flow of example 1, assume that at $M_1 = 0.1$, we have $P_1 = 600kPa$ and $T_1 = 450$ K. At section 2, further downstream, $M_2 = 0.5$. Calculate (a) P_2, (b) T_2 , (c) C_2, and (d) P_{02}.

Solution:

As preliminary information, we can calculate C_1 and P_{01} from the given data:

$$C_1 = M_1 a_1 = 0.1[(1.4)(287)(450)]^{1/2} = 0.1(425) = 42.5m/s$$

$$P_{01} = P_1(1+0.2M_1^2)^{3.5} = (600.10^3)[1+0.2(0.1)^2]^{3.5} = 604kPa$$

Join the table now to locate the following reports.

Section	M	p/p^*	T/T^*	V/V^*	p_0/p_0^*
1	0.1	10.9435	1.1976	0.1094	5.8218
2	0.5	2.1381	1.1429	0.5345	1.3399

Utilize these proportions to calculate all downstream properties:

$$P_2 = P_1\frac{P_2/P^*}{P_1/P^*} = (600.10^3)\frac{2.1381}{10.9435} = 117kPa$$

$$T_2 = T_1\frac{T_2/T}{T_1/T^*} = (450)\frac{1.1429}{1.1976} = 429k$$

$$C_2 = C\frac{C_2/C^*}{C_1/C^*} = (42.5)\frac{0.5345}{0.1094} = 208m/s$$

$$P_{02} = P_{01}\frac{P_{02}/P_0^*}{P_{01}/P_0^*} = (604.10^3)\frac{1.3399}{5.8218} = 139kPa$$

Part 2: Compressible Flow in Pipes with Heat Transfer and No Friction (RAYLEIGH Theory)

V-1 Analysis of Rayleigh flow and fundamental equations

The addition or removal of heat has a notable effect on a compressible flow. Here, we restrict the analysis to the flow with heat transfer without friction in a conduit of constant cross-section.

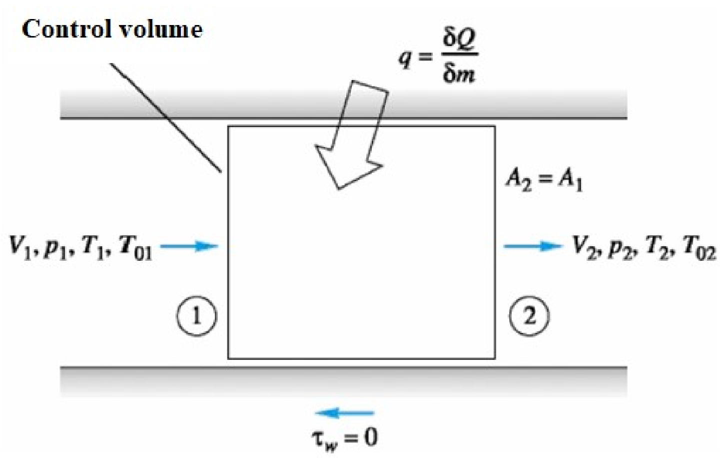


Figure V.2 : Elementary control volume for a frictionless flow in a constant cross-section pipe with heat transfer.

The length of this element is indeterminate in this theory.

Consider the elementary control volume of the conduit in figure (V.1). Between sections 1 and 2, a quantity of heat δQ is added (or removed) to each elemental mass δm passing through. In the absence of friction or changes in section, the conservation relationships for the control volume are quite straightforward:

Continuity : $$\rho_1 C_1 = \rho_2 C_2 = G = const \quad \text{(V.18)}$$

Quantity of motion with respect to x: $$\rho_1 C = \rho_2 C_2 = G = const \quad \text{(V.19)}$$

$$P_1 - P_2 = G(C_2 - C_1)$$

Energy :
$$\dot{Q} = \dot{m}\left(h_2 + \frac{1}{2}V_2^2 - h_1 - \frac{1}{2}V_1^2\right) \tag{V.20}$$

Where
$$q = \frac{\dot{Q}}{\dot{m}} = \frac{\delta Q}{\delta m} = h_{02} - h_{01} \tag{V.21}$$

The transfer of heat results in a change in the stagnation enthalpy of the flow. We will not specify exactly how the heat is transferred (whether through combustion, nuclear reaction, evaporation, condensation, or wall heat exchange), but we will simply state that it occurred in the quantity Q between 1 and 2. However, we note that this wall heat exchange is not a suitable candidate for the theory because wall convection is inevitably associated with wall friction, which we have overlooked.

To complete the analysis, we utilize the ideal gas relations and the Mach number.

$$\left.\begin{aligned} &\frac{P_2}{\rho_2 T_2} = \frac{P_1}{\rho_1 T_1} \qquad h_{02} - h_{01} = cp\ \ (T_{02} - T_{01}) \\ &\frac{C_2}{C_1} = \frac{M_2 a_2}{M_1 a_1} = \frac{M_2}{M_1}\left(\frac{T_2}{T_1}\right)^{1/2} \end{aligned}\right\} \tag{V.22}$$

For a given amount of transferred heat $q = \delta Q / \delta M$ or, for a specified variation $h_{02} - h_{01}$, equations (V.18) and (V.19) can be solved algebraically to determine the ratios of properties P_2 / P_1, M_2 / M_1, etc., between the inlet and the outlet. It is important to note that because heat transfer allows entropy to either increase or decrease, the second law of thermodynamics does not impose any restrictions on these solutions.

Before discussing these functions of property ratios, we illustrate the effect of heat transfer in figure (V.19), which depicts T_0 and T as a function of the Mach number in the duct. Heating raises T0 while cooling lowers it. The maximum possible value of T_0 occurs at $M = 1.0$, and we observe that heating, whether the inlet is subsonic or supersonic, drives the Mach number of the duct towards unity. This is analogous to the effect of friction discussed in the previous chapter. The temperature of the ideal gas increases from M = 0 to $M = 1/\gamma^{1/2}$ and then decreases. Thus, there exists a peculiarity or an unexpected region where heating (with T_0 increasing) actually reduces the temperature of the gas, the difference being reflected in a significant increase in the kinetic energy of the gas. For $\gamma = 1.4$, this particular region is found between M = 0.845 and M = 1.0 (interesting information, but not very useful)

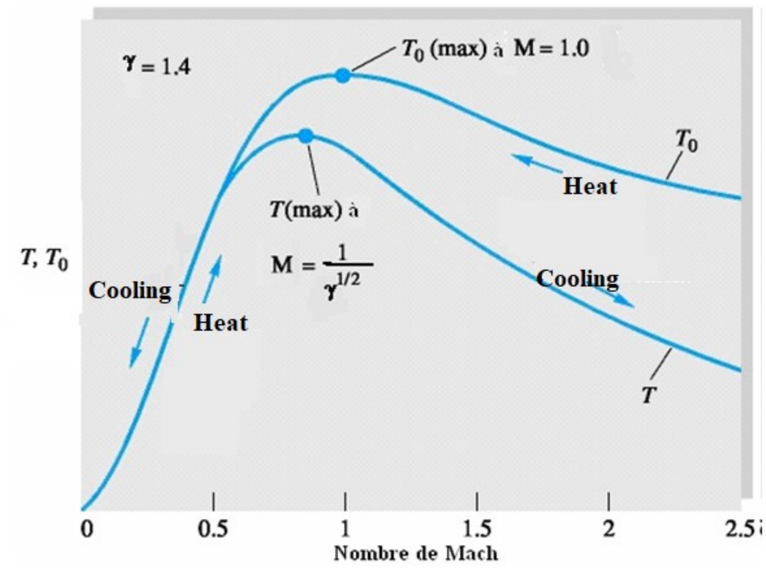


Figure V.3 The impact of heat transfer on the Mach number.

The complete list of the effects of a simple change in T_0 on the internal flow properties of the pipeline is as follows:

	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
T_0	Increases	Increases	Decreases	Decreases
M	Increases	Decreases	Decreases	Increases
p	Decreases	Increases	Increases	Decreases
ρ	Decreases	Increases	Increases	Decreases
V	Increases	Decreases	Decreases	Increases
p_0	Decreases	Decreases	Increases	Increases
T	☪	Increases	☆	Decreases
Entropy	Increases	Increases	Decreases	Decreases

☪ : Increase until $M = 1/\gamma^{1/2}$ and then decrease.

☆ : Decrease until $M = 1/\gamma^{1/2}$ and then decrease.

The most significant item on this list is likely the stagnation pressure P_0, which consistently decreases during heating when the flow is subsonic or supersonic. Therefore, heating increases the Mach number of a flow, but results in a loss of effective pressure.

V.2 Variation of flow characteristics as a function of the Mach number

The equations (V.18) and (V.19) can be rearranged in terms of the Mach number, and the results are tabulated. For convenience, we specify the output section as sonic, M=1, with reference properties $T_0^*, T^*, P^*, \rho^*,\ V^*\ and\ P_0^*$. The input is assumed to be at an arbitrary Mach number M. The equations (V.18) and (V.19) then take the following form:

$$\frac{T_0}{T_0^*} = \frac{(\gamma+1)M^2[2 + (\gamma-1)M^2]}{(1+\gamma M^2)^2} \tag{V.23}$$

$$\frac{T}{T^*} = \frac{(\gamma+1)^2 M^2}{(1+\gamma M^2)^2} \tag{V.24}$$

$$\frac{P}{P^*} = \frac{\gamma+1}{1+\gamma M^2} \tag{V.25}$$

$$\frac{C}{C^*} = \frac{\rho^*}{\rho} = \frac{(\gamma+1)\ M^2}{1+\gamma M^2} \tag{V.26}$$

$$\frac{P_0}{P_0^*} = \frac{\gamma+1}{1+\gamma M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\gamma/(\gamma-1)} \tag{V.27}$$

These formulas are all tabulated according to the Mach number in table (III). The tables are quite convenient when input properties such as M_1, V_1, etc., are provided; however, they become somewhat cumbersome when the information pertains to T_{01} and T_{02}. Below is an example that illustrates this.

V.3 Variation of entropy

Entropy, a measure of disorder or randomness in a system, plays a significant role in Rayleigh Flow. In an adiabatic process, where no heat is exchanged with the surroundings, the entropy of a perfect gas remains constant. However, in Rayleigh Flow, the addition or removal of heat alters the entropy. This change can be quantified using the formula:

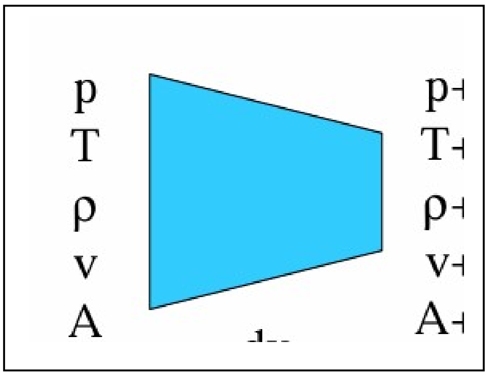
$$\boldsymbol{\Delta s = C_P\, ln\left(\frac{T_2}{T_1}\right) - r ln(\frac{P_2}{P_1})} \tag{V.28}$$

PART3 : Flow with friction and heat exchange.

This section discusses heat transfer and skin friction in turbulent pipe flow with variable physical properties. The constant properties solution has been considered only so far as is necessary for the flow and heat transfer analysis with variable physical properties.

So far we have examined (1-d) isentropic flows with area change

1. Isentropic
2. reversible (e.g., no friction): $P_s=0$
3. adiabatic (no heat addition or loss): Q=0



Mach Number Equation

Combine conservation, state equations–can

Algebraically show

$$\frac{dM^2}{M^2} = \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\left\{\underbrace{\frac{\delta q}{Cp\,T_0}}_{HEAT\,TRANSFER}(1+\gamma\,M^2)+\gamma M^2\underbrace{\frac{\lambda dx}{D}}_{FRICTION}-2\frac{dA}{A}\right\}$$

So we have three ways to change M of flow

Area change (dA): previously studied

Friction: λ> 0, same effect as dA

Heat transfer:heating, δq>0, like ($-dA$) cooling,δq< 0

Exercises with solutions

EXERCISES RELATED TO CHAPTER I

Exercise 01

What is the Mach number of an aircraft flying at sea level at 0°C at a speed of:

a) 1440km/h ?

b) 900km/h ?.

Response :

a)331.19m/s ; M=1.2 ; b) M=0.75

Exercise 02

A military aircraft can fly at Mach 2. What is its speed in kilometers per hour if it is flying at sea level at 0°C?

Response :

C=2384.7km/h

Exercise 03

Imagine that you visit a world where the atmosphere is composed of hydrogen. Your spacecraft can travel at Mach 20 on Earth, as evidenced by a sound measured at 5°C. On this new planet, what is your Mach number if the speed of sound in hydrogen is 1267 m/s?

Response :

M=5.27

Exercise 04

An aircraft is flying at a Mach number of M= 0.95 and at an altitude where the atmospheric pressure is P_{atm} = 0,2332 bar and the density ρ = 0,349 Kg/m^3.

1) Calculate the speed of the airplane in km/h.

2) Calculate the pressure and temperature at the stagnation point on the leading edge of the wing.

The air is considered to be an ideal gas: γ = 1,4 et r = 287 J/kg.K

Solution :

1) $V = M.a = M.\sqrt{\frac{\gamma P}{\rho}}$ A.N : $V = \frac{290{,}56\ m}{s} = \frac{1046{,}02\ km}{h}$

2) $P_0 = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1} .P$

A.N: $\boxed{P_0 = \left(1 + \frac{1.4-1}{2} (0{,}95)^2\right)^{1.4/1.4-1} .\ 0{,}2332 = 0{,}416\, Bar}$

$T_0 = \left(1 + \frac{\gamma-1}{2} M^2\right).T$

Or $r.T = \frac{P}{\rho}$ (air considéré gaz parfait) $\Rightarrow T = \frac{P}{\rho r}$

$\Rightarrow T = \frac{0{,}2332}{(287).(0{,}349)}$=232,82K

$$T_0 = \left(1 + \frac{1.4-1}{2}\ 0{,}95^2\right).(232{,}82) = 4168.05 Pa$$

Exercise 05

A celestial body in free fall, slowed down by the layers of air in the upper atmosphere, descends to Earth. At an altitude of 10 km :

- the speed of the body V=3000m/s, the air temperature T=223*k* the density of air ρ=0,412*kg /m^3*, the pressure of air P=0,265 bar we provide. γ = 1,4.

Requested work:

1) Calculate the speed of sound a.

2) Determine the Mach number M.

3) What is the nature of the airflow around the body?

4) Apply Saint-Venant's theorem to calculate the temperature T0 and the pressure P0 of the air at the stagnation point.

Solution

1) 1) Speed of sound: $a = \sqrt{\gamma.\frac{P}{\rho}}$ A.N : $a = \sqrt{1{,}4.\frac{26500}{0{,}412}} = 300m/s$

2) 2) Density of mass: $M = \frac{C}{a}$ A.N : $M = \frac{3000}{300} = 10$

3) $M > 1$ Thus, the flow is supersonic..

Stagnation Temperature $T_0 = \left(1 + \frac{\gamma-1}{2}\, M^2\right).T$

A.N : $T_0 = \left(1 + \frac{1.4-1}{2}\,(10)^2\right).223 = 532K$

4) 4) Stagnation pressure : $P_0 = \left(1 + \frac{\gamma-1}{2}\, M^2\right)^{\gamma/\gamma-1}.P$

A.N: $P_0 = \left(1 + \frac{1{,}4-1}{2}\,(10)^2\right)^{1.4/1.4-1}$ .26500=11246 Pa

EXERCISES RELATED TO CHAPTER II

Exercise1

The air from a reservoir flows into an adiabatic and frictionless duct. The temperature and pressure in the reservoir are T = 400 K and P = 500 kPa.

Determine M, T, ρ, and V at a section of the duct where P = 430 kPa.

Solution :

Mach Number :

$$\frac{P_0}{P_S} = \left(1 + \frac{\gamma - 1}{2} M_S^2\right)^{\gamma/\gamma-1}$$

$$\Rightarrow M_S = \sqrt{\frac{2}{\gamma - 1}\left(\left(\frac{P_0}{P_S}\right)^{\gamma-1/\gamma} - 1\right)} = 0{,}468$$

The temperature T

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^{2} \Rightarrow T = \frac{T_0}{1 + \frac{\gamma - 1}{2} M^{2}}$$

$$\Rightarrow T = 383{,}21\ k$$

Density $\boldsymbol{\rho}$

$$\frac{P}{\rho} = r\,T \Rightarrow \rho = \frac{P}{r\,T} = \frac{430.10^3}{(287)\,(383{,}21)} = 3{,}9\ \ kg/m^3$$

The flow velocity C

$$M = \frac{C}{a} \Rightarrow C = M\ .a = M\ .\sqrt{\gamma\, r\, T}$$

Numerical application: $C = 0{,}468\ \sqrt{(1{,}4)\,(287)\,(383.21)} = 183{,}64 m/s$

$$\boxed{C = 184.64 m/s}$$

Exercise 2

Calculate the stagnation parameters (P_0 and T_0) for an air flow at speeds of C=50, 100, and 250 m/s, with the static parameters of the air being: P=0, T=300K.

Determine the relative deviations between:

1) The stagnation pressures and the static pressures;

2) The stagnation temperatures and the static temperatures.

Response :

1) 1.28%, 5.6%, 41% 2) 0.33%, 1.68%, 10.37%.

Exercise3

The combustion chamber pressure is P_0=50.65 x 10^4 Pa and T_0=220 K. Determine the critical pressure, critical temperature, and critical velocity.

Solution :

Calculation of critical pressure, critical temperature, and critical velocity

$$\frac{P_0}{P_c} = \left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1} \Rightarrow P_C = \frac{P_0}{\left(\frac{\gamma+1}{2}\right)^{\gamma/\gamma-1}}$$

N.A : $P_C = \frac{50{,}65\ 10^4}{1{,}2^{3.5}} = 26{,}75\ 10^4 Pa$

$$\frac{T_0}{T_c} = \frac{\gamma+1}{2} \Rightarrow T_c = \frac{2\,T_0}{\gamma+1}$$

N.A :

$$T_c = \frac{(2\,).\ (400)}{2{,}4} = 183.33k$$

$$C_c = a_c = \sqrt{\gamma\, r\, T_c} = \sqrt{(1{,}4)(287)(333{,}34)} = 271.40 m/s$$

Exercise 4

A reservoir contains compressed air at a pressure of P_0 = 4 bar, which is assumed to be the stagnation pressure at the initial state. The opening of a valve in this reservoir causes the air to expand outward in the form of a jet with a diameter of d = 5 mm.

The external parameters of the air jet at the final state are:

- Pressure P = 1 bar,
- Temperature T = 25°C,

Given γ = 1.4 and r = 287 J/Kg.K.

1) Calculate the sound velocity "a" outside the reservoir in (m/s).

2) Determine the density ρ of the air outside the reservoir in (kg/m^3).

(Assuming that air behaves as an ideal gas)

3) Write the Saint-Venant equation in terms of the pressure ratio between a stagnation point and a point on the air jet.

4) From this, deduce the Mach number M at the level of the air jet.

5) What is the nature of the flow?

6) Calculate the flow velocity C of the air jet in (m/s).

7) From this, deduce the mass flow rate $\dot{m}$ (kg/s).

Solution :

1) 1) Sound velocity: $a = \sqrt{\gamma . r . T}$ A.N : $a = \sqrt{(1,4).(287).(298)} = 346 m/s$

2) Density : $\rho = \frac{P}{r.T}$ A.N : $\rho = \frac{10^5}{287.298} = 1,169\ Kg/m^3$

3) Saint- venant equation: $1 + \frac{\gamma-1}{2} M^2 = \left(\frac{P_0}{P}\right)^{\gamma-1/\gamma}$

4) Mach Number : $M = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_0}{P}\right)^{\gamma-1/\gamma} - 1\right]} = 1,558$

5) $M > 1$ therefore, the flow is supersonic.

6) Velocity : C= M. a A.N : C=(1,558)(346)=306 m/s

7) Mass flow rate: $\dot{m} = \rho . A . C = \rho . \frac{\pi . d^2}{4} . C$

N.A : $\dot{m} = 1,169 . \frac{\pi . (0,005)^2}{4} . 306 = \frac{0.0123 kg}{s}$

Exercise 5

Compressed air from a large reservoir escapes to the outside through an orifice at a Mach number of M=0.77. The expansion occurs in the atmosphere where the pressure is P=Patm=1.013 bar.

The ratio of specific heats is given as: γ =1.4 and r=287 J/Kg.K.

Required work:

1) By applying the Saint-Venant equation, determine the pressure P0 (in bar) inside the reservoir.

2) From what pressure P_0 does the flow become supersonic.

Solution :

1) Saint-Venant equation : $1 + \frac{\gamma-1}{2} M^2 = \left(\frac{P_0}{P}\right)^{\gamma-1/\gamma}$ therefore

$$P_0 = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\gamma/\gamma-1} . P$$

N.A : $P_0 = \left(1 + \frac{1.4-1}{2} (0,77)^2\right)^{1.4/1.4-1} .1,014 . 10^5 = 150097,21\ Pa = 1,5\ Bar$

2) $M > 1 \Rightarrow P_i > P . \left(\frac{\gamma+1}{2}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}$

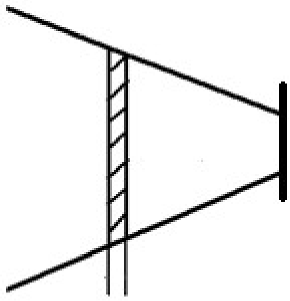
N.A : $P_i > 1,014 . 10^5 . \left(\frac{1,4+1}{2}\right)^{\left(\frac{1,4}{1,4-1}\right)} = 191943 Pa \approx 2\ Bar$

EXERCISES RELATED TO CHAPTER II-Parte2

Exercise 1

The air, under a pressure of 10 bar and at a temperature of 27°C, flows through a converging nozzle towards a region where the pressure is 6 bar. Calculate the parameters of the air at the exit section and the mass flow rate, given that the exit diameter is 20 mm.

Solution :



$$\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/\gamma-1}$$

$$\frac{P_0}{P} = \frac{10}{6} = 1{,}66 < 1{,}89$$

Therefore, it can be stated that in this output section, the critical conditions are not met because

$$\frac{P_0}{P} < 1{,}89$$

$$\frac{T_S}{T_0} = \left(\frac{P_S}{P_0}\right)^{\gamma-1/\gamma} \Rightarrow T_S = T_0.\left(\frac{P_S}{P_0}\right)^{\gamma-1/\gamma}$$

$$\boxed{\Rightarrow T_S = 300.\left(\frac{6}{10}\right)^{0{,}4/1{,}4} = 259{,}26K}$$

$$\frac{\rho_S}{\rho_0} = \left(\frac{T_S}{T_0}\right)^{1/\gamma-1} \Rightarrow \rho_S = \rho_0.\left(\frac{T_S}{T_0}\right)^{1/\gamma-1}$$

$$\frac{P_0}{\rho_0} = r\,T_0 \Rightarrow \rho_0 = \frac{P_0}{r\,T_0} = \frac{10.10^5}{(287).(300)} = 11{,}6 \quad kg/m^3$$

N.A: $\boxed{\rho_S = 11{,}6.\left(\frac{259{,}26}{300}\right)^{1/0.4}}$

$$\boxed{\rho_S = 8{,}063\ kg/m^3}$$

Calculation of velocity C_S:

$$\frac{T_0}{T_S} = 1 + \frac{\gamma - 1}{2} {M_S}^2 \Rightarrow M_S = \sqrt{\frac{2}{\gamma - 1}\left(\frac{T_0}{T_S} - 1\right)}$$

A.N: $M_S = \sqrt{\frac{2}{0,4}\left(\frac{300}{259,26} - 1\right)} = 0,886$

$$\boxed{M_S = 0,886}$$

$$M_S = \frac{C_S}{a_S} \Rightarrow C_S = M_S . a_S = M_S . \sqrt{\gamma\, r\, T_s}$$

A.N: $C_S = 0,886\ \sqrt{(1,4)\ (287)\ (259,26)}\ = 285,96\, m/s$

$$\boxed{C_S = 285,96\, m/s}$$

Calculation of mass flow rate at the outlet:

$$\dot{m}_s = \rho_S . C_S . A_S = (8,063\ )\ (285,96)\ (3,14\ \ 10^{-4}) = 0,72\, kg/s$$

$$\boxed{\dot{m}_S = 0,72\, kg/s}$$

Exercise 2

We consider a converging-diverging nozzle where the section at point 1, upstream of the throat, has the following characteristics: area A_1 = 13.4 cm²; p_1 = 2 bar; T_1 = 30°C; and C_1 = 174.77 m/s.

a/ Determine the speed a1 of sound at section 1.

b/ Determine the pressure and temperature at the generating conditions.

c/ What should be the area of the section at the throat for the nozzle to be initiated? What should be the maximum value of the downstream pressure for the nozzle to be suitable if the exit section is A_S = 15.51 cm²? Given: r = 287 J/(kg·K) and γ =1,4.

Solution :

a-Calculation of sonic velocity a_1

$a_1 = \sqrt{\gamma\, r\, T_1} = \sqrt{(1,4)(\ 287)(303)} = 348,92 m/s$

b- Calculation of P_0 , T_0

$$M_1 = \frac{C_1}{a_1} = \frac{174,77}{348,92} = 0,5$$

$$\frac{P_0}{P_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/_{\gamma-1}} \Rightarrow P_0 = P_1 \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/_{\gamma-1}}$$

$$\Rightarrow P_0 = 2.10^5 \left(1 + \frac{0,4}{2}(0,5)^2\right)^{1.4/_{0.4}}$$

$$P_0 = 2,372.10^5\ Pa = 2,372 bar$$

$$\frac{T_0}{T_1} = \left(\frac{P_0}{P_1}\right)^{\gamma-1/_{\gamma}} \Rightarrow T_0 = T_1 . \left(\frac{P_0}{P_1}\right)^{\gamma-1/_{\gamma}}$$

$$\boxed{\Rightarrow T_0 = 303. \left(\frac{2.372}{2}\right)^{0.4/_{1.4}} = 318.15K}$$

c- The critical section A_C

$$\boxed{\frac{A_1}{A_c} = \frac{1}{M_1} . \left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_1^2\right)\right]^{\gamma+1/_{2(\gamma-1)}}}$$

$$A_c = \frac{A_1}{\frac{1}{M_1} . \left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_1^2\right)\right]^{\gamma+1/_{2(\gamma-1)}}}$$

$$A_c = \frac{13,4 \quad 10^{-4}}{\frac{1}{0,5} . \left[\frac{2}{2,4}\left(1 + 0,2\ (0,5)^2\ \right)\right]^3} = 10 \ \ 10^{-4}\ m^2 = 10\ cm^2$$

In order for the nozzle to be initiated, it is necessary for Ac to equal 10 cm^2

d) The outlet section As

$$\frac{A_s}{A_c} = \frac{15,51}{10} = 1,551$$

According to the table, this report corresponds to a Mach number equal to $M_s = 1,9$

Therefore:

$$\frac{P_0}{P_s} = \left(1 + \frac{\gamma - 1}{2} M_S^2\right)^{\gamma/_{\gamma-1}}$$

$$P_s = \frac{P_0}{\left(1 + \frac{\gamma - 1}{2} M_S^2\right)^{\gamma/_{\gamma-1}}}$$

N.A : $P_s = 0{,}353\ \ 10^5\ Pa = 0{,}353\ bar$ or according to the table $\frac{P_s}{P_c} = 0{,}1490 \Rightarrow\ P_s = (0{,}1490).\, P_0 = 0{,}353\ bar$

Exercise 3

A Laval insulated nozzle is suitable for the pressure ratio $P/P_i = 0.149$. The flowing gas is air (r = 287 J/(kg·K) and γ = 1.405). Calculate:

a/ The diameter of its throat if the exit section area is As = 7.07 cm²;

b/ The mass flow rate under the following generating conditions: P_0 = 10 bar and T_0 = 450 K;

c/ The upper limit pressure PLS required for the nozzle to be primed.

Solution :

Calculation of the critical section $\boldsymbol{A_c}$

$$\frac{P_s}{P_c} = 0{,}1490$$

$$\frac{P_0}{P_s} = \left(1 + \frac{\gamma - 1}{2} M_S^2\right)^{\gamma/\gamma-1} = \frac{1}{0{,}149}$$

$$M_S = \sqrt{\frac{2}{\gamma - 1}\left(\left(\frac{P_0}{P_S}\right)^{\gamma-1/\gamma} - 1\right)} = 1{,}898 \simeq 1{,}9$$

$$\boxed{M_S = 1{,}9}$$

$$\boxed{\frac{A_s}{A_c} = \frac{1}{M_s}.\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_s^2\right)\right]^{\gamma+1/2(\gamma-1)}}$$

$$A_c = \frac{A_s}{\frac{1}{M_s}.\left[\frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M_s^2\right)\right]^{\gamma+1/2(\gamma-1)}}$$

$$\boxed{A_c = 4{,}55\ \ 10^{-4} m^2}$$

Calculation of mass flow rate at the outlet:

$$\dot{m}_s = \rho_S . C_S . A_S$$

Calculation of C_S

$$M_S = \frac{C_S}{a_S} \Rightarrow C_S = M_S . a_S = M_S . \sqrt{\gamma \, r \, T_s}$$

$$\frac{T_S}{T_0} = \left(\frac{P_s}{P_0}\right)^{\gamma-1/\gamma} \Rightarrow T_S = T_0 . \left(\frac{P_s}{P_0}\right)^{\gamma-1/\gamma}$$

$$\boxed{\Rightarrow T_S = (450).(0,149)^{0.4/1.4} = 261,56K}$$

$$\boxed{C_S = 615,9m/s}$$

$$\frac{P_s}{\rho_s} = r \, T_s \Rightarrow \rho_s = \frac{P_s}{r \, T_s} = \frac{(0,149).(10 \, . \, 10^5)}{(287).(261,56)} = 1,98 \;\; kg/m^3$$

N.A: $\boxed{\rho_S = 1,98kg/m^3}$

$$\boxed{\dot{m}_S = \rho_S . C_S . A_S = 0,862 \; kg/s}$$

PLS=P_c

$$\boxed{\frac{P_c}{P_0} = 0,528 \Rightarrow P_c = 0,528 \, . \, P_0 = 5,28 \, Bar}$$

Exercise 5

A convergent-divergent insulated nozzle has a ratio of 2 between the area of the exit section As and the area of the throat section A_C. The air enters the nozzle at a stagnation pressure P_0 = 10 bar and a stagnation temperature T_0 = 360 K. The area of the throat section is A_C = 500 mm^2. Determine:

The mass flow rate, pressure, temperature, Mach number, and velocity at the exit of the nozzle in the following two cases:

a/ The velocity at the throat is sonic and the divergent section functions as a nozzle;

b/ The velocity at the throat is sonic and the divergent section functions as a diffuser.

Solution :

a) Divergent nozzle case

$$\frac{A_s}{A_c} = 2$$

Verify the line $\frac{A_s}{A_c} = 2$ The table's supersonic section is due to the flow regime within the divergent nozzle being supersonic. This leads to :

$\frac{A_s}{A_c} = 1{,}997$ $\frac{T_s}{T_0} = 0{,}505$ $M_s = 2{,}2$ $\frac{P_s}{P_0} = 0{,}0934$

$\rightarrow \frac{A_s}{A_c} = 2$ $M_s = ?$

$\frac{A_s}{A_c} = 2{,}392$ $\frac{T_s}{T_0} = 0{,}4616$ $M_s = 2{,}4$ $\frac{P_s}{P_0} = 0{,}0684$

$$\frac{2 - 1{,}997}{M_s - 2{,}2} = \frac{2{,}392 - 1{,}997}{2{,}4 - 2{,}2}$$

$$\boxed{M_s = 2{,}201}$$

$$\frac{T_0}{T_S} = 1 + \frac{\gamma - 1}{2} M_S{}^2 \Rightarrow T_S = \frac{T_0}{1 + \frac{\gamma - 1}{2} M_S{}^2}$$

$$\boxed{\Rightarrow T_S = 182{,}845k}$$

$$\frac{P_S}{P_0} = \left(\frac{T_s}{T}\right)^{\gamma/\gamma-1} \Rightarrow P_S = P_0.\left(\frac{T_s}{T}\right)^{\gamma/\gamma-1}$$

$$\boxed{\Rightarrow P_S = 0{,}933\text{bar}}$$

$$\frac{\rho_S}{\rho_0} = \left(\frac{T_s}{T}\right)^{1/\gamma-1} \Rightarrow \rho_S = \rho_0.\left(\frac{T_s}{T}\right)^{1/\gamma-1}$$

$$\boxed{\Rightarrow \rho_S = 9{,}67 kg/m^3}$$

Calculation of C_S

$$M_S = \frac{C_S}{a_S} \Rightarrow C_S = M_S . a_S = M_S . \sqrt{\gamma\, r\, T_s}$$

$$\boxed{\Rightarrow C_S = 596{,}57 m/s}$$

Calculation of mass flow rate

$$\boxed{\dot{m}_S = \rho_S . C_S . A_S = 1{,}061\ kg/s}$$

b) The canalization acts as a diffuser.

$$\frac{A_s}{A_c} = 2$$

Verify the line $\frac{A_s}{A_c} = 2$ the subsonic part of the table is due to the flow regime in the divergent diffuser being subsonic. It is derived from:

$$\frac{A_s}{A_c} = 2{,}097 \qquad M_s = 0{,}29$$

$$\rightarrow \frac{A_s}{A_c} = 2 \qquad M_s = ?$$

$$\frac{A_s}{A_c} = 2{,}034 \qquad M_s = 0{,}30$$

$$\frac{2 - 2{,}097}{M_s - 0{,}29} = \frac{2{,}034 - 2{,}097}{0{,}3 - 0{,}29}$$

$$\boxed{M_s = 0{,}305}$$

$$\frac{T_0}{T_S} = 1 + \frac{\gamma - 1}{2} {M_S}^2 \Rightarrow T_S = \frac{T_0}{1 + \frac{\gamma - 1}{2} {M_S}^2}$$

$$\boxed{\Rightarrow T_S = 353{,}4k}$$

$$\frac{P_S}{P_0} = \left(\frac{T_S}{T}\right)^{\gamma/\gamma - 1} \Rightarrow P_S = P_0 . \left(\frac{T_S}{T}\right)^{\gamma/\gamma - 1}$$

$$\boxed{\Rightarrow P_S = 9{,}374 \text{ bar}}$$

$$\frac{P_s}{\rho_s} = r\, T_s \Rightarrow \rho_s = \frac{P_s}{r\, T_s}$$

$$\boxed{\Rightarrow \rho_S = 9{,}241 kg/m^3}$$

Calculation of C_S

$$M_S = \frac{C_S}{a_S} \Rightarrow C_S = M_S . a_S = M_S . \sqrt{\gamma\, r\, T_s}$$

$$\boxed{\Rightarrow C_S = 114{,}93 m/s}$$

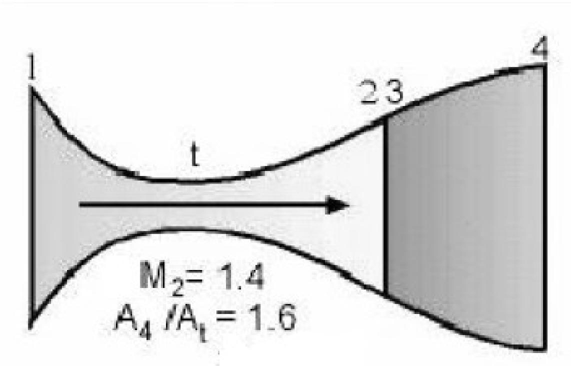
Calculation of mass flow rate

$$\boxed{\dot{m}_s = \rho_S . C_S . A_S = 1{,}061\ kg/s}$$

EXERCISES RELATED TO CHAPTER III-Part1

Exercise 1

The air is supplied through a converging-diverging nozzle, which has a throat with a specific area ratio. A_4/A_t as illustrated in the following figure.



The air coming from a reservoir at a pressure of $P = 2MPa\ and\ T = 400K$. A normal shock occurs in the divergent nozzle at a section where $P_{01} = P_{02} = 2MPa$ the upstream Mach number is 1.4. Determine:

1) The Mach number downstream of the shock wave
2) The Mach number at the exit
3) The pressure at the exit
4) The temperature at the exit.

Solution

The flow from the reservoir, section 1, is isentropic until the normal shock at section 2, and from section 3, just downstream of the normal shock to the exit at section 4. The stagnation temperatures remain constant throughout the flow and across the normal shock wave: $T_{01}=T_{02}=T_{03}=T_{04}=400K$. The stagnation pressures do not change in an isentropic flow, $P_{01} = P_{02} = 2MPa$ and $P_{03} = P_{04}$, However, the stagnation pressures vary across the shock., $P_{02} > P_{03}$.

Based on the Mach number at section 2 and utilizing isentropic relations, we have:

$$\frac{A_2}{A_t} = \frac{A_3}{A_t} = \frac{A_2}{A_t^*} = \frac{1}{M_2}\ \frac{(1 + 0.2\ M_2^2)^3}{1.728} = 1.115$$

Normal shock relations can be applied through the shock. Therefore,

1)

$$M_3 = \left[\frac{(\gamma - 1)M^2\ + 2}{2\gamma M_2^2 - (\gamma - 1)}\right]^{1/2} = \left[\frac{(0.4)(1.4)^2\ + 2}{2(1.4)(1.4)^2\ - (0.4)}\right]^{1/2} = 0.740$$

2) By continuing the analysis through the shock, one has:

$$P_{04} = P_{03} = P_{02}\ \left[\frac{(\gamma + 1)M_2^2}{2 + (\gamma - 1)M_2^2}\right]^{\gamma/(\gamma-1)} \left[\frac{\gamma + 1}{2\gamma M_2^2 - (\gamma - 1)}\right]^{1/(\gamma-1)}$$

$$P_{04} = P_{03} = 2\ \left[\frac{(2.4)(1.4)^{1.2}}{2 + (0.4)(1.4)^2}\right]^{3.5} \left[\frac{2.4}{2(1.4)(1.4)^2\ - (0.4)}\right]^{2.5} = 1.92MPa$$

$$\frac{A_3^*}{A_2^*} = \frac{M_3}{M_2}\left[\frac{2 + (\gamma - 1)M_2^2}{2 + (\gamma - 1)M_3^2}\right]^{-(\gamma+1)/2(\gamma-1)} = \frac{0.74}{1.4}\ \left[\frac{2 + (0.4)(1.4)^2}{2 + (0.4)(0.74)^2}\right]^{2.4/0.8} = 1.044$$

We are aware of the report A_4/A_t, The flow remains isentropic between sections 3 and 4. We can formulate an expression for the ratio of the area between the outlet section and the throat of the nozzle. :

$$\frac{A_4}{A_t} = 1.6 = \frac{A_4}{A_4^*}\ .\frac{A_4^*}{A_3^*}.\frac{A_3^*}{A_2^*}.\frac{A_2^*}{A_t} = \frac{A_4}{A_4^*}(1)(1.044)(1.115)$$

In addressing the issue of solving for $\frac{A_4}{A_4^*}$ we obtain $\frac{A_4}{A_4^*} = 1.374$

By utilizing the previously developed equation for choked and isentropic flow, we can express:

$$\frac{A_4}{A_4^*} = 1.374 = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma-1}\right]^{(\gamma+1)/2(\gamma-1)}$$

$where \qquad 1.374 = \frac{1}{M_4}\left[\frac{2+(0.4){M_4}^2}{0.4}\right]^3$

We find:

$$M_4 = 0.483$$

3) En connaissant Ma_4, nous pouvons continuer en utilisant les relations isentropiques, nous trouvons :

$$P_4 = \frac{2\,P_{04}}{[2+(\gamma-1)M_4^2]^{\gamma/\gamma-1}} = \frac{2\times 1.92\ \times 10^6}{[2+(0.4)(0.483)^2\,]^{3.5}} = 1.637MPa$$

4)

$$T_4 = \frac{2\,T_{04}}{[2+(\gamma-1)M_4^2]} = \frac{2\times 400}{[2+(0.4)(0.483)^2\,]} = 382K$$

Exercise 2

A converging/diverging nozzle with a throat diameter of d_c=1cm draws in air under standard atmospheric conditions (101325Pa and 15 °C) with an upstream velocity of zero. The diameter of the divergent section of the nozzle increases linearly in the axial direction until reaching an exit diameter of 2cm, and the length of the nozzle from its throat to the exit is 10 cm.

1) Determine the mass flow rate required for the nozzle to be primed.

2) Calculate the pressure P_j and the Mach number M_j at the exit of the flow for a suitable jet to develop downstream of the nozzle.

3) Calculate the pressure P_j and the Mach number M_j at the exit of the flow in the scenario where a normal shock is present in the nozzle 5cm downstream of the throat.

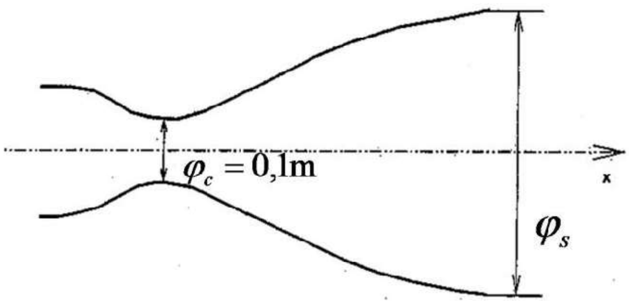
Response:

1) $\dot{m} = 0.0187\text{kg/s}$ 2) $P_j = 3{,}02kPa, M_j = 2{,}934$ 3) $P_j = 35{,}4kPa, M_j = 0{,}867$

Exercise 3

We consider a converging-diverging nozzle such that $\frac{\varphi_s}{\varphi_c} = 2{,}605$ and $\varphi_c = 0{,}1m$. It is designed to provide a uniform supersonic flow at the outlet for the adaptation (inlet nozzle). The generating conditions are then: P_i=10 bars T_i=600K

We provide γ=1,4; r=287 J/kg.K



1) Assuming the nozzle is fully choked, calculate M_s, P_s, and T_s.

2) What will be the value of the static exit pressure P_s if a normal shock occurs in the exit plane (S)? Then calculate M_s and T_s downstream of this shock.

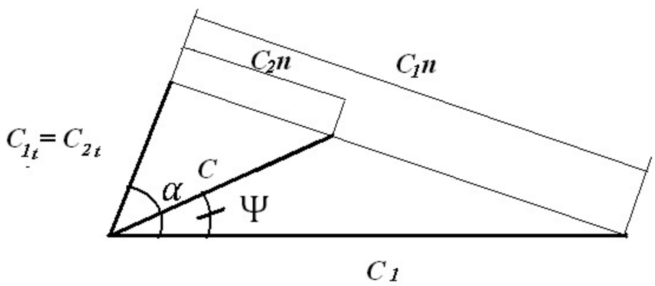
3) If the shock is positioned at $\varphi_{ch} = 0{,}155m$, Determine the parameters characterizing the flow: (M_1,P_1,T_1) before the shock, (M_2,P_2,T_2) after the shock, and (M_s,P_s,T_s) at the exit.

4) The static pressure at the exit is now P_s=2.588 bars. Provide the diameter of the section where the shock occurs, specifying the Mach number, static pressure, and temperature before the shock, after the shock, and at the exit.

EXERCISES RELATED TO CHAPTER III-Part2

Exercise1

Consider a flow of the type depicted in the following figure (V.1), where the upstream Mach number is M_1=2, featuring an oblique wave with an angle $\alpha = 64°$.



$$tg\Psi = 2ctg64° \frac{1 + 0.2 \times 4 \times \sin^2 64° - 1}{4(1.4 + \cos 128°) + 2} = 1.423, \qquad \Psi = 23°$$

The number of downstream Mach is given by

$$M_2^2 sin^2 41° = \frac{1 + 0.2 \times 4 \times \sin^2 64°}{4 \times 1.4 + \sin^2 64° + 2}, \qquad M_2 = 0.942$$

Result approximately corresponding to the minimum of M_1 on the curve Ψ=23° in figure (V.1)

$$\frac{P_2}{P_1} = \frac{2.8}{2.4} \times 4 \times \sin^2 64° - \frac{0.4}{2.4} = 3.60$$

$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma - 1}{2} {M_1}^2}{1 + \frac{\gamma - 1}{2} {M_2}^2} = \frac{1 + 0.2 \times 4}{1 + 0.2 + 0.942^2} = 1.528$$

If the evolution were isentropic, we would have had

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} = 3.6^{0.286} = 1.442$$

The isentropic efficiency of this compression is therefore:

$$\eta_s = \frac{\frac{T_{2s}}{T_1} - 1}{\frac{T_2}{T_1} - 1} = \frac{0.442}{0.528} = 0.837$$

For the straight wave studied previously, we had $M_1 = 2, \ M_2 = 0.577$

$$\frac{P_2}{P_1} = 4.5, \quad \frac{T_2}{T_1} = \frac{1 + 0.2 \times 4}{1 + 0.2 \times 0.577^2} = 1.688, \quad \frac{T_{2s}}{T_1} = 4.5^{0.286} = 1.537, \quad \eta_s = \frac{0.537}{0.688} = 0.78$$

EXERCISES RELATED TO CHAPTER IV

Exercise

An airflow (ideal gas, γ=1.4) passes through a supersonic nozzle. At a certain point on the wall, the geometry forms a convex angle of θ=20∘. The Mach number before the expansion is M=1.5.

Questions:

1. Calculate the value of the Prandtl-Mayer function $\nu(M)$ in degrees.
2. Deduce the value of $\nu(M_2)$.
3. Find the final Mach number M_2.

4. Calculate the following isentropic ratios: $\frac{P_2}{P_1}, \frac{T_2}{T_1}, \frac{\rho_2}{\rho_1}$

Solution

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - tan^{-1}\sqrt{(M^2-1)}$$

$M_1 = 1.5 \quad \gamma = 1.4:$

$\frac{\gamma+1}{\gamma-1} = \frac{2.4}{0.4} = 6$

$M_1^2 - 1 = 1.5^2 - 1 = 2.25 - 1 = 1.25$

$\frac{\gamma-1}{\gamma+1} = \frac{0.4}{2.4} \approx 0.1667$

$$\nu(M_1) = \sqrt{6} \cdot \tan^{-1}\left(\sqrt{0.1667 \cdot 1.25}\right) - \tan^{-1}\left(\sqrt{1.25}\right)$$

$$\nu(M_1) \approx 2.449 \cdot \tan^{-1}(0.456) - \tan^{-1}(1.118)$$

$$\nu(M_1) \approx 2.449 \cdot 24.56° - 48.01° \approx 60.11° - 48.01° = 12.10°$$

$$v(M_1) \approx 12.10°$$

2/

$$v(M_2) = v(M_1) + \theta = 12.10° + 20° = 32.10°$$

3/ $M_2 \approx 2.25$

4/

$$\frac{P_2}{P_1} = \left(\frac{1+0.2 \cdot M_1^2}{1+0.2 \cdot M_2^2}\right)^{\frac{1.4}{0.4}} = \left(\frac{1+0.2 \cdot 2.25}{1+0.2 \cdot 5.06}\right)^{3.5}$$

$$= \left(\frac{1.45}{2.012}\right)^{3.5} \approx (0.7207)^{3.5} \approx 0.337$$

$$\frac{T_2}{T_1} = \frac{1+0.2 \cdot M_1^2}{1+0.2 \cdot M_2^2} = \frac{1.45}{2.012} \approx 0.7207$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{0.337}{0.7207} \approx 0.4675$$

EXERCISES RELATED TO CHAPTER V-Part1

Issue

A constant cross-section duct serves as the site for an adiabatic airflow. At the entrance, the conditions are $M_1 = 0.3$, P_1 =3 Bars, $T_1 = 300K$.

At the exit, the Mach number is $M_2 = 0.7$.

1. Examine this flow by comparing direct calculations with the use of Fanno tables?
2. Calculate the pressure at the outlet?

3. Determine the stagnation pressures at both the inlet and outlet?
4. Calculate the conditions P*, P_0^*, T* at the fictitious point where M=1?
5. Calculate the velocities at the inlet and outlet?
6. Calculate the length of the pipe if λ=0.04=Cte, D=0.1m?

Solution

1)

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2} = \frac{1.018}{1.098} = 0.927$$

$$T_2 = 300 \times 0.927 = 278k$$

The reading of Fanno's tables provides:

For :

$\begin{cases} M_1 = 0.3,\ T_1/T^* = 1.178, \\ M_2 = 0.7,\ T_2/T^* = 1.0929, \end{cases}$ From where $\frac{T_2}{T_1} = \frac{1.0929}{1.1788} = 0.927$

2) Calculation of the output pressure.

The continuity equation is expressed as:

$\rho_1 C_1 = \rho_2 C_2$ Ou $\frac{P_1}{r\,T_1} M_1 \sqrt{\gamma\, r\, T_1} = \frac{P_2}{r\,T_2} M_2 \sqrt{\gamma\, r\, T_2}$

From where:

$\frac{P_1}{P_2} = \frac{M_2}{M_1}\sqrt{\frac{T_1}{T_2}} = \frac{0.7}{0.3}\sqrt{\frac{1}{0.927}} = 2.423 \qquad P_2 = \frac{3}{2.423} = 1.239Bar$

The reading of Fanno's tables provides:

$\begin{cases} for \quad M_1 = 0.3 \quad {P_1}/{P^*} = 3.619 \\ for \quad M_2 = 0.7 \quad {P_2}/{P^*} = 1.493 \end{cases}$ d'où $\frac{P_1}{P_2} = \frac{3.619}{1.493} = 2.42$

3) Calculation of the stagnation pressures at the inlet and outlet.

$${P_{01}}/{P_1} = \left(1+\frac{\gamma-1}{2}M_1^2\right)^{\gamma/\gamma-1} = 1.018^{3.5} = 1.0644$$

$${P_{02}}/{P_2} = \left(1+\frac{\gamma-1}{2}M_2^2\right)^{\gamma/\gamma-1} = 1.098^{3.5} = 1.387$$

$$P_{01} = 3.1932\ Bar$$

$$P_{02} = 1.238 \times 1.387 = 1.717\ Bar$$

The use of Fanno tables necessitates the prior calculation of conditions P^*, P_0^*, T^* at the point of flow where M=1 (a fictitious point in this example).

3) 3) Calculation of the conditions P^*, P_0^*, T^*, at the fictitious point where M=1.

From the reading of the tables, we derive:

$$P^* = \frac{P_1}{3.619} = \frac{P_2}{1.493} = 0.829\ Bar$$

$$T^* = \frac{T_1}{1.178} = \frac{T_2}{1.0929} = 254.5K$$

The Barré de Saint-Venant theorem provides

$$P_0^*/_{P^*} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/_{\gamma-1}} = 1.2^{3.5} = 1.892$$

From where we verify by reading the tables,

$$P_{01}^* = 2.035\ P_0^* = 2.035 \times 1.57 = 3.193 Bar$$

$$P_{02}^* = 1.094\ P_0^* = 1.094 \times 1.57 = 1.717 Bar$$

4) Calculation of speeds at the entrance and exit

One can either utilize the definition of the Mach number at each point $C = M\sqrt{\gamma\ r\ T}$, one should utilize the Barré de Saint Venant theorem.

$$C_p T_i = C_p T + \frac{C^2}{2}$$

We will employ this latter method, allowing the reader to verify the results by using the Mach number.

The generating temperature is provided by

$$\frac{T_i}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2 \qquad T_i = 300 \times 1.018 = 305.4k$$

$$C_1 = \sqrt{2C_p(T_i - T_1)} = \sqrt{200 \times 5.4} = 104 m/s$$

$$C_2 = \sqrt{2C_p(T_i - T_2)} = \sqrt{200 \times 27.4} = 234 m/s$$

4) 4) Calculation of the length of the pipe if $\lambda = 0.04 = C^{te}, D = 0.1m$

The reading of Fanno tables provides

$$\frac{\lambda}{D} L_1^* = 5.299, \qquad \frac{\lambda}{D} L_2^* = 0.208$$

From where

$$L = L_1^* - L_2^* = \frac{D}{\lambda}(5.299 - 0.208) = \frac{0.1}{0.04} \times 5.091 = 12.73$$

EXERCISES RELATED TO CHAPTER V-Part2

Exercise 1

A mixture of air and fuel, assumed to have a specific heat ratio γ = 1.4, enters a combustion chamber with an inlet velocity of $C_1 = 75$ m/s, pressure $P_1 = 150$kPa, and temperature $T_1 = 300$K. The heat added through combustion is 900 kJ/kg of the mixture. Calculate (a) the exit properties C_2, P_2, and T_2, and (b) the total amount of heat added that would have resulted in a sonic exit flow.

Solution:

(a) (a) First, we calculate $T_{01} = T_1 + \frac{C_1^2}{2Cp} = 300 + \frac{(75)^2}{[2(1005)]} = 303k$. Thus, we calculate the variation in the stagnation temperature of the gas :

$$q = Cp(T_{02} - T_{01})$$

Where $T_{02} = T_{01} + \frac{q}{Cp} = 303 + \frac{900000}{1005} = 1199K$

We have sufficient information to calculate the initial Mach number. :

$$a_1 = \sqrt{\gamma\, r\, T_1} = [(1{,}4)(287)(300)]^{1/2} = \frac{347m}{s} \qquad M_1 = \frac{V_1}{a_1} = \frac{75}{347} = 0{,}216$$

For this Mach number, we use equation (VII.3)a or table 4 to determine the sonic value T_0^* :

A $M_1 = 0{,}216 : \frac{T_{01}}{T_0^*} \approx 0{,}1992 \quad ou \quad T_0^* = \frac{303}{0.1992} \approx 1521k$

The stagnation temperature ratio at section 2 is $T_{02} / T_0^* = 1199 / 1521 = 0.788$, which corresponds to a Mach number $M_2 \approx 0.573$ in table (III).

We will now use table (III) at M_1 and M_2 to tabulate the desired property ratios..

Section	M	V / V^*	p / p^*	T / T^*
1	0.216	0.1051	2.2528	0.2368
2	0.573	0.5398	1.6442	0.8876

The output properties are determined by using these proportions to identify state 2 from state 1:

$$C_2 = C\,\frac{C_2/C^*}{C_1/C^*} = (75)\frac{0{,}5398}{0{,}1051} = 385m/s$$

$$P_2 = P_1\,\frac{P_2/P^*}{P_1/P^*} = (150.10^3)\frac{1{,}6442}{2{,}2528} = 109kPa$$

$$T_2 = T_1 \frac{T_2/T}{T_1/T^*} = (300)\frac{0,8876}{0,2368} = 1124k$$

(b) The addition of the maximum allowable heat would lead the exit Mach number to the unit.:

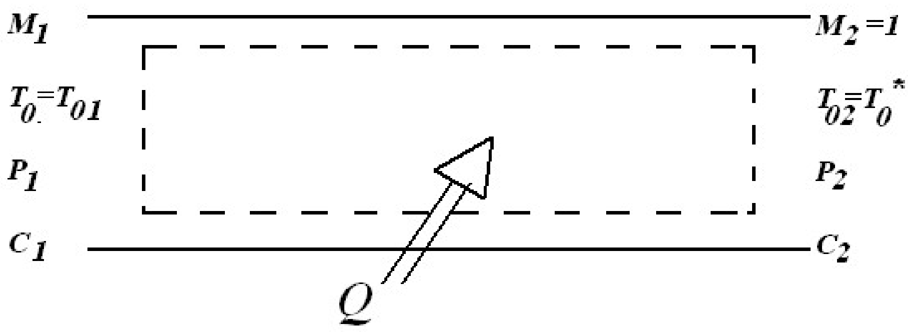
$$T_{02} = T_0^* = 1521k$$

$$q_{max} = C_p(T_0^* - T_{01}) = (1005)(1521 - 303) \approx 1,22 \times 10^6 J/Kg$$

Exercise 2

Consider a cylindrical post-combustion chamber. The inlet stop temperature is 1000K, while the outlet temperature is 2500K. What could be the maximum Mach number at the inlet? ($\gamma = 1.4$)

Solution



$$\begin{cases} T_{01} = 1000k \\ T_{02} = 2500k \end{cases}$$

Therefore $\frac{T_{01}}{T_{02}} = 0.4$

We are looking for M_1, assuming that M_2=1.

The table of the Rayleigh curve:

M	T_0/T_0^*	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
0.32	0.3837	2.0991	0.4512	4.6523	1.1904
0.34	0.4206	2.0657	0.4933	4.1877	1.1822

Therefore, according to the Rayleigh table and by employing interpolation between two values $0.32 < M_1 < 0.34$

One can find $M_1 = \left(\frac{0.32-0.34}{0.3837-0.4206}\right)(0.4 - 0.4206) + 0.34 = 0.33$

Appendix: Tables of Compressible Flow

TABLE I of compressible flows $\gamma = 1.405$

$\mathcal{M}$	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{S}{S_c}$	c ($T_0 = 288$ °K)
0,10	0,993 0	0,995	0,998 0	5,820	33,4
0,11	0,991 6	0,994	0,997 5	5,303	36,9
0,12	0,990 0	0,993	0,997 0	4,870	40,4
0,13	0,988 3	0,992	0,996 5	4,503	43,8
0,14	0,986 4	0,991	0,995 9	4,188	47,2
0,15	0,984 4	0,990	0,995 3	3,915	50,6
0,16	0,982 3	0,988	0,994 7	3,676	54,0
0,17	0,980 0	0,986	0,994 1	3,465	57,4
0,18	0,977 6	0,984	0,933 4	3,278	60,8
0,19	0,975 1	0,982	0,992 7	3,111	64,2
0,20	0,972 4	0,980	0,992 0	2,961	67,6
0,21	0,969 6	0,978	0,991 2	2,826	71,0
0,22	0,966 7	0,976	0,990 3	2,704	74,4
0,23	0,963 7	0,974	0,989 4	2,594	77,8
0,24	0,960 6	0,972	0,988 5	2,494	81,2
0,25	0,957 3	0,970	0,987 5	2,402	84,6
0,26	0,953 9	0,968	0,986 5	2,317	87,9
0,27	0,950 4	0,965	0,985 4	2,238	91,2
0,28	0,946 8	0,962	0,984 3	2,165	94,5
0,29	0,943 1	0,959	0,983 2	2,097	97,8
0,30	0,939 3	0,956	0,982 0	2,034	101,1
0,31	0,935 4	0,953	0,980 8	1,975	104,4
0,32	0,931 4	0,950	0,979 6	1,920	107,7
0,33	0,927 2	0,947	0,978 4	1,869	111,0
0,34	0,922 9	0,944	0,977 1	1,821	114,3
0,35	0,918 5	0,941	0,975 8	1,776	117,6
0,36	0,914 0	0,938	0,974 4	1,734	120,9
0,37	0,909 5	0,935	0,973 0	1,695	124,2
0,38	0,904 9	0,932	0,971 6	1,658	127,5
0,39	0,900 2	0,928	0,970 1	1,623	130,7
0,40	0,895 3	0,924	0,968 6	1,590	133,9
0,41	0,890 4	0,920	0,967 0	1,559	137,1
0,42	0,885 4	0,916	0,965 4	1,529	140,3
0,43	0,880 3	0,912	0,963 8	1,501	143,5
0,44	0,875 1	0,908	0,962 2	1,474	146,7
0,45	0,869,9	0,904	0,960 5	1,448	149,9
0,46	0,864 6	0,900	0,958 8	1,424	153,1
0,47	0,859 2	0,896	0,957 1	1,401	156,3
0,48	0,853 7	0,892	0,955 4	1,379	159,5
0,49	0,848 1	0,888	0,953 6	1,359	162,7
0,50	0,842 5	0,884	0,951 8	1,340	165,9
0,51	0,836 8	0,880	0,950 0	1,322	169,1
0,52	0,831 1	0,876	0,948 1	1,304	172,2
0,53	0,825 3	0,872	0,946 2	1,287	175,3
0,54	0,819 5	0,868	0,944 3	1,271	178,4
0,55	0,813 6	0,864	0,942 3	1,255	181,5

$\mathcal{M}$	$\frac{p}{p_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	$\frac{S}{S_c}$	c ($T_0 = 288$ °K)
0,56	0,807 7	0,860	0,940 3	1,240 0	184,6
0,57	0,801 7	0,856	0,938 3	1,226 0	187,7
0,58	0,795 6	0,851	0,936 3	1,212 6	190,8
0,59	0,789 5	0,846	0,934 2	1,199 8	193,9
0,60	0,783 4	0,841	0,932 1	1,187 6	197,0
0,61	0,777 2	0,836	0,930 0	1,176 0	200,0
0,62	0,771 0	0,831	0,927 9	1,165 0	203,0
0,63	0,764 8	0,826	0,925 8	1,154 5	206,0
0,64	0,758 5	0,821	0,923 6	1,144 5	209,0
0,65	0,752 2	0,816	0,921 4	1,135 0	212,0
0,66	0,745 9	0,811	0,919 2	1,125 9	215,0
0,67	0,739 5	0,806	0,916 9	1,117 3	218,0
0,68	0,733 1	0,801	0,914 6	1,109 1	221,0
0,69	0,726 7	0,796	0,912 3	1,101 3	224,0
0,70	0,720 2	0,791	0,909 9	1,093 9	227,0
0,71	0,713 8	0,786	0,907 5	1,086 9	229,9
0,72	0,707 3	0,781	0,905 1	1,080 3	232,8
0,73	0,700 8	0,776	0,902 7	1,074 0	235,7
0,74	0,694 3	0,771	0,900 3	1,068 0	238,6
0,75	0,687 8	0,766	0,897 9	1,062 3	241,5
0,76	0,681 3	0,761	0,895 4	1,056 9	244,4
0,77	0,674 8	0,756	0,892 9	1,051 8	247,3
0,78	0,668 3	0,751	0,890 4	1,047 0	250,2
0,79	0,661 8	0,746	0,887 9	1,042 5	253,1
0,80	0,655 3	0,741	0,885 4	1,038 2	255,9
0,81	0,649	0,736	0,882 8	1,034 2	258,7
0,82	0,642	0,730	0,880 2	1,030 4	261,5
0,83	0,636	0,724	0,877 6	1,026 9	264,3
0,84	0,629	0,718	0,875 0	1,023 7	267,1
0,85	0,623	0,712	0,872 4	1,020 7	269,9
0,86	0,616	0,707	0,869 8	1,017 9	272,7
0,87	0,610	0,702	0,867 2	1,015 3	275,5
0,88	0,603	0,697	0,864 5	1,012 9	278,3
0,89	0,597	0,692	0,861 7	1,010 7	281,0
0,90	0,590	0,687	0,859 0	1,008 7	283,7
0,91	0,584	0,681	0,856 3	1,006 9	286,4
0,92	0,577	0,675	0,853 6	1,005 4	289,1
0,93	0,571	0,670	0,850 9	1,004 1	291,8
0,94	0,565	0,665	0,848 2	1,003 0	294,5
0,95	0,559	0,660	0,845 4	1,002 1	297,2
0,96	0,553	0,655	0,842 7	1,001 4	299,9
0,97	0,547	0,650	0,839 9	1,000 8	302,5
0,98	0,540	0,645	0,837 2	1,000 4	305,1
0,99	0,534	0,640	0,834 4	1,000 2	307,7
1,00	0,528	0,635	0,831 6	1,000 0	310,3

$\mathcal{M}$	$\frac{p}{p_0}$	$\frac{p'_0}{p_0}$	$\frac{p_2}{p_1}$	$\frac{\rho}{\rho_0}$	$\frac{\rho_2}{\rho_1}$	$\frac{T}{T_0}$	$\frac{S}{S_c}$	c ($T_0 = 288$ °K)
1,00	0,527 5	1,000 00	1,000	0,634 3	1,000 0	0,831 6	1,000 0	310,3
1,01	0,521 3	0,999 99	1,023	0,629 0	1,016 7	0,828 8	1,000 2	312,9
1,02	0,515 2	0,999 98	1,047	0,623 7	1,033 4	0,826 0	1,000 5	315,5
1,03	0,509 1	0,999 94	1,071	0,618 4	1,050 1	0,823 1	1,000 9	318,0
1,04	0,503 0	0,999 88	1,095	0,613 2	1,066 9	0,820 3	1,001 5	320,6
1,05	0,497 0	0,999 8	1,119	0,608 0	1,083 7	0,817 5	1,002 2	323,1
1,06	0,491 0	0,999 7	1,144	0,602 8	1,100 6	0,814 6	1,003 1	325,6
1,07	0,485 1	0,999 5	1,169	0,597 6	1,117 5	0,811 8	1,004 1	328,1
1,08	0,479 2	0,999 4	1,194	0,592 4	1,134 5	0,808 9	1,005 2	330,6
1,09	0,473 3	0,999 2	1,219	0,587 2	1,151 5	0,806 1	1,006 5	333,1
1,10	0,467 5	0,998 9	1,245	0,582 1	1,168 5	0,803 2	1,007 9	335,5
1,11	0,461 7	0,998 5	1,271	0,577 0	1,185 6	0,800 3	1,009 5	338,0
1,12	0,456 0	0,998 1	1,297	0,571 9	1,202 7	0,797 5	1,011 2	340,4
1,13	0,450 3	0,997 6	1,323	0,565 8	1,219 8	0,794 6	1,013 1	342,8
1,14	0,444 6	0,997 1	1,350	0,561 7	1,237 0	0,791 7	1,015 1	345,2
1,15	0,439 0	0,996 5	1,377	0,556 6	1,254 2	0,788 8	1,017 3	347,6
1,16	0,433 4	0,995 9	1,404	0,551 6	1,251 4	0,785 9	1,019 6	350,0
1,17	0,427 9	0,995 2	1,431	0,546 8	1,288 7	0,783 0	1,022 0	352,3
1,18	0,422 4	0,994 4	1,458	0,541 6	1,306 0	0,780 1	1,024 6	354,7
1,19	0,417 0	0,993 6	1,486	0,536 6	1,323 3	0,777 2	1,027 3	357,0
1,20	0,411 6	0,992 8	1,514	0,531 7	1,340 6	0,774 3	1,030 1	359,3
1,21	0,406 3	0,992	1,542	0,526 8	1,357 9	0,771 4	1,033 1	361,6
1,22	0,401 0	0,991	1,570	0,521 9	1,375 2	0,768 5	1,036 2	363,9
1,23	0,395 7	0,990	1,599	0,517 0	1,392 5	0,765 6	1,039 4	366,2
1,24	0,390 5	0,989	1,628	0,512 2	1,409 8	0,762 7	1,042 8	368,5
1,25	0,385 3	0,987	1,657	0,507 4	1,427 1	0,759 7	1,046 3	370,8
1,26	0,380 2	0,986	1,686	0,502 6	1,444 4	0,756 8	1,050 0	373,0
1,27	0,375 1	0,985	1,716	0,497 8	1,461 8	0,753 9	1,053 8	375,2
1,28	0,370 1	0,983	1,746	0,493 0	1,479 2	0,751 0	1,057 7	377,4
1,29	0,365 1	0,982	1,776	0,488 3	1,496 6	0,748 0	1,061 7	379,6
1,30	0,360 2	0,980	1,806	0,483 6	1,514 0	0,745 1	1,065 9	381,8
1,31	0,355 3	0,978	1,836	0,478 9	1,531 3	0,742 2	1,070 2	384,0
1,32	0,350 5	0,976	1,867	0,474 3	1,548 7	0,739 3	1,074 6	386,2
1,33	0,345 7	0,974	1,898	0,469 7	1,566 0	0,736 3	1,079 2	388,4
1,34	0,341 0	0,972	1,929	0,465 1	1,583 3	0,733 4	1,083 9	390,5
1,35	0,336 3	0,970	1,961	0,460 5	1,600 6	0,730 5	1,088 7	392,6
1,36	0,331 7	0,968	1,993	0,456 0	1,618 0	0,727 6	1,093 6	394,7
1,37	0,327 1	0,966	2,025	0,451 5	1,635 3	0,724 7	1,098 7	396,8
1,38	0,322 5	0,963	2,057	0,447 0	1,652 6	0,721 7	1,103 9	398,9
1,39	0,318 0	0,961	2,089	0,442 5	1,669 9	0,718 8	1,109 2	401,0

$\mathcal{M}$	$\frac{p}{p_0}$	$\frac{p'_0}{p_0}$	$\frac{p_2}{p_1}$	$\frac{\rho}{\rho_0}$	$\frac{\rho_2}{\rho_1}$	$\frac{T}{T_0}$	$\frac{S}{S_c}$	c ($T_0 = 288$ °K)
1,40	0,313 6	0,958	2,122	0,438 1	1,687 2	0,715 9	1,114 7	403,1
1,50	0,271 9	0,929	2,461	0,395 7	1,859	0,687 0	1,176	423
1,60	0,234 8	0,894	2,823	0,356 6	2,028	0,658 6	1,249	443
1,70	0,202 2	0,855	3,208	0,320 6	2,193	0,630 8	1,336	461
1,80	0,173 7	0,813	3,617	0,287 8	2,353	0,603 8	1,436	477
1,90	0,149 0	0,769	4,049	0,258 0	2,508	0,577 7	1,551	492
2,00	0,127 7	0,723	4,505	0,231 1	2,657	0,552 5	1,682	507
2,20	0,093 4	0,630	5,486	0,185 1	2,939	0,505 0	1,997	533
2,40	0,068 4	0,542	6,561	0,148 3	3,197	0,461 6	2,392	556
2,60	0,050 2	0,462	7,30	0,118 9	3,431	0,422 2	2,879	576
2,80	0,036 9	0,392	8,992	0,095 6	3,644	0,386 5	3,474	594
3,00	0,027 3	0,330	10,347	0,077 1	3,835	0,354 4	4,19	608
3,50	0,013 2	0,215	14,14	0,046 0	4,232	0,287 3	6,70	640
4,00	0,006 66	0,141	18,53	0,028 24	4,538	0,235 8	10,53	663
4,50	0,003 51	0,092 9	23,49	0,017 90	4,774	0,196 1	16,21	680
5,00	0,001 927	0,063 1	29,04	0,011 68	4,959	0,165 0	24,38	693
5,50	0,001 100	0,043 5	35,17	0,007 83	5,105	0,140 3	35,81	702
6,00	0,000 651	0,030 6	41,89	0,005 39	5,222	0,120 6	51,45	711
7,00	0,000 250	0,015 9	57,08	0,002 73	5,395	0,091 7	100,00	723
8,00	0,000 107	0,008 87	74,61	0,001 49	5,513	0,071 6	181,4	730

The indices referenced are those utilized in the course, specifically:

"0" for generating conditions

"c" for critical conditions

"1" and "2" for conditions respectively before and after the right shock

P'_0 : denotes the generating pressure following the shock.

TABLE II: Adiabatic flow with friction at constant section). (Fanno curves).

M	$\bar{f}L^* / D$	p / p^*	T / T^*	ρ / ρ^* V^* / V	p_0 / p_0^*
0.00	∞	∞	1.2000	∞	∞
0.02	1778.4500	54.7701	1.1999	45.6454	28.9421
0.04	440.3523	27.3817	1.1996	22.8254	14.4815
0.06	193.0311	18.2509	1.1991	15.2200	9.6659
0.08	106.7182	13.6843	1.1985	11.4182	7.2616
0.10	66.9216	10.9435	1.1976	9.1378	5.8218
0.12	45.4080	9.1156	1.1966	7.6182	4.8643
0.14	32.5113	7.8093	1.1953	6.5333	4.1824
0.16	24.1978	6.8291	1.1939	5.7200	3.6727
0.18	18.5427	6.0662	1.1923	5.0879	3.2779
0.20	14.5333	5.4554	1.1905	4.5826	2.9635
0.22	11.5961	4.9554	1.1885	4.1694	2.7076
0.24	9.3865	4.5383	1.1863	3.8255	2.4956
0.26	7.6876	4.1851	1.1840	3.5347	2.3173
0.28	6.3572	3.8820	1.1815	3.2857	2.1656
0.30	5.2993	3.6191	1.1788	3.0702	2.0351
0.32	4.4467	3.3887	1.1759	2.8818	1.9219
0.34	3.7520	3.1853	1.1729	2.7158	1.8229
0.36	3.1801	3.0042	1.1697	2.5684	1.7358
0.38	2.7054	2.8420	1.1663	2.4367	1.6587
0.40	2.3085	2.6958	1.1628	2.3184	1.5901
0.42	1.9744	2.5634	1.1591	2.2115	1.5289
0.44	1.6915	2.4428	1.1553	2.1145	1.4740
0.46	1.4509	2.3326	1.1513	2.0261	1.4246
0.48	1.2453	2.2313	1.1471	1.9451	1.3801
0.50	1.0691	2.1381	1.1429	1.8708	1.3398
0.52	0.9174	2.0519	1.1384	1.8024	1.3034
0.54	0.7866	1.9719	1.1339	1.7391	1.2703
0.56	0.6736	1.8975	1.1292	1.6805	1.2403
0.58	0.5757	1.8282	1.1244	1.6260	1.2130
0.60	0.4908	1.7634	1.1194	1.5753	1.1882
0.62	0.4172	1.7026	1.1143	1.5279	1.1656
0.64	0.3533	1.6456	1.1091	1.4836	1.1451
0.66	0.2979	1.5919	1.1038	1.4421	1.1265
0.68	0.2498	1.5413	1.0984	1.4032	1.1097
0.70	0.2081	1.4935	1.0929	1.3665	1.0944
0.72	0.1721	1.4482	1.0873	1.3320	1.0806
0.74	0.1411	1.4054	1.0815	1.2994	1.0681
0.76	0.1145	1.3647	1.0757	1.2686	1.0570
0.78	0.0917	1.3261	1.0698	1.2395	1.0471
0.80	0.0723	1.2893	1.0638	1.2119	1.0382
0.82	0.0559	1.2542	1.0578	1.1858	1.0305
0.84	0.0423	1.2208	1.0516	1.1609	1.0237
0.86	0.0310	1.1889	1.0454	1.1373	1.0179

M	$\bar{f}L^*/D$	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
0.88	0.0218	1.1583	1.0391	1.1148	1.0129
0.90	0.0145	1.1291	1.0327	1.0934	1.0089
0.92	0.0089	1.1011	1.0263	1.0730	1.0056
0.94	0.0048	1.0743	1.0198	1.0535	1.0031
0.96	0.0021	1.0485	1.0132	1.0348	1.0014
0.98	0.0005	1.0238	1.0066	1.0170	1.0003
1.00	0.0000	1.0000	1.0000	1.0000	1.0000
1.02	0.0005	0.9771	0.9933	0.9837	1.0003
1.04	0.0018	0.9551	0.9866	0.9681	1.0013
1.06	0.0038	0.9338	0.9798	0.9531	1.0029
1.08	0.0066	0.9133	0.9730	0.9387	1.0051
1.10	0.0099	0.8936	0.9662	0.9249	1.0079
1.12	0.0138	0.8745	0.9593	0.9116	1.0113
1.14	0.0182	0.8561	0.9524	0.8988	1.0153
1.16	0.0230	0.8383	0.9455	0.8865	1.0198
1.18	0.0281	0.8210	0.9386	0.8747	1.0248
1.20	0.0336	0.8044	0.9317	0.8633	1.0304
1.22	0.0394	0.7882	0.9247	0.8524	1.0366
1.24	0.0455	0.7726	0.9178	0.8418	1.0432
1.26	0.0517	0.7574	0.9108	0.8316	1.0504
1.28	0.0582	0.7427	0.9038	0.8218	1.0581
1.30	0.0648	0.7285	0.8969	0.8123	1.0663
1.32	0.0716	0.7147	0.8899	0.8031	1.0750
1.34	0.0785	0.7012	0.8829	0.7942	1.0842
1.36	0.0855	0.6882	0.8760	0.7856	1.0940
1.38	0.0926	0.6755	0.8690	0.7773	1.1042
1.40	0.0997	0.6632	0.8621	0.7693	1.1149
1.42	0.1069	0.6512	0.8551	0.7615	1.1262
1.44	0.1142	0.6396	0.8482	0.7540	1.1379
1.46	0.1215	0.6282	0.8413	0.7467	1.1501
1.48	0.1288	0.6172	0.8344	0.7397	1.1629
1.50	0.1360	0.6065	0.8276	0.7328	1.1762
1.52	0.1433	0.5960	0.8207	0.7262	1.1899
1.54	0.1506	0.5858	0.8139	0.7198	1.2042
1.56	0.1579	0.5759	0.8071	0.7135	1.2190
1.58	0.1651	0.5662	0.8004	0.7074	1.2344
1.60	0.1724	0.5568	0.7937	0.7016	1.2502
1.62	0.1795	0.5476	0.7869	0.6958	1.2666
1.64	0.1867	0.5386	0.7803	0.6903	1.2836
1.66	0.1938	0.5299	0.7736	0.6849	1.3010
1.68	0.2008	0.5213	0.7670	0.6796	1.3190
1.70	0.2078	0.5130	0.7605	0.6745	1.3376
1.72	0.2147	0.5048	0.7539	0.6696	1.3567
1.74	0.2216	0.4969	0.7474	0.6648	1.3764
1.76	0.2284	0.4891	0.7410	0.6601	1.3967
1.78	0.2352	0.4815	0.7345	0.6555	1.4175
1.80	0.2419	0.4741	0.7282	0.6511	1.4390
1.82	0.2485	0.4668	0.7218	0.6467	1.4610
1.84	0.2551	0.4597	0.7155	0.6425	1.4836
1.86	0.2616	0.4528	0.7093	0.6384	1.5069
1.88	0.2680	0.4460	0.7030	0.6344	1.5308
1.90	0.2743	0.4394	0.6969	0.6305	1.5553
1.92	0.2806	0.4329	0.6907	0.6267	1.5804

M	$\bar{f}L^*/D$	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
1.94	0.2868	0.4265	0.6847	0.6230	1.6062
1.96	0.2929	0.4203	0.6786	0.6193	1.6326
1.98	0.2990	0.4142	0.6726	0.6158	1.6597
2.00	0.3050	0.4082	0.6667	0.6124	1.6875
2.02	0.3109	0.4024	0.6608	0.6090	1.7160
2.04	0.3168	0.3967	0.6549	0.6057	1.7451
2.06	0.3225	0.3911	0.6491	0.6025	1.7750
2.08	0.3282	0.3856	0.6433	0.5994	1.8056
2.10	0.3339	0.3802	0.6376	0.5963	1.8369
2.12	0.3394	0.3750	0.6320	0.5934	1.8690
2.14	0.3449	0.3698	0.6263	0.5905	1.9018
2.16	0.3503	0.3648	0.6208	0.5876	1.9354
2.18	0.3556	0.3598	0.6152	0.5848	1.9698
2.20	0.3609	0.3549	0.6098	0.5821	2.0050
2.22	0.3661	0.3502	0.6043	0.5794	2.0409
2.24	0.3712	0.3455	0.5989	0.5768	2.0777
2.26	0.3763	0.3409	0.5936	0.5743	2.1153
2.28	0.3813	0.3364	0.5883	0.5718	2.1538
2.30	0.3862	0.3320	0.5831	0.5694	2.1931
2.32	0.3911	0.3277	0.5779	0.5670	2.2333
2.34	0.3959	0.3234	0.5728	0.5647	2.2744
2.36	0.4006	0.3193	0.5677	0.5624	2.3164
2.38	0.4053	0.3152	0.5626	0.5602	2.3593
2.40	0.4099	0.3111	0.5576	0.5580	2.4031
2.42	0.4144	0.3072	0.5527	0.5558	2.4479
2.44	0.4189	0.3033	0.5478	0.5537	2.4936
2.46	0.4233	0.2995	0.5429	0.5517	2.5403
2.48	0.4277	0.2958	0.5381	0.5497	2.5880
2.50	0.4320	0.2921	0.5333	0.5477	2.6367
2.52	0.4362	0.2885	0.5286	0.5458	2.6864
2.54	0.4404	0.2850	0.5239	0.5439	2.7372
2.56	0.4445	0.2815	0.5193	0.5421	2.7891
2.58	0.4486	0.2781	0.5147	0.5402	2.8420
2.60	0.4526	0.2747	0.5102	0.5385	2.8960
2.62	0.4565	0.2714	0.5057	0.5367	2.9511
2.64	0.4604	0.2682	0.5013	0.5350	3.0073
2.66	0.4643	0.2650	0.4969	0.5333	3.0647
2.68	0.4681	0.2619	0.4925	0.5317	3.1233
2.70	0.4718	0.2588	0.4882	0.5301	3.1830
2.72	0.4755	0.2558	0.4839	0.5285	3.2439
2.74	0.4791	0.2528	0.4797	0.5269	3.3061
2.76	0.4827	0.2498	0.4755	0.5254	3.3695
2.78	0.4863	0.2470	0.4714	0.5239	3.4342
2.80	0.4898	0.2441	0.4673	0.5225	3.5001
2.82	0.4932	0.2414	0.4632	0.5210	3.5674
2.84	0.4966	0.2386	0.4592	0.5196	3.6359
2.86	0.5000	0.2359	0.4552	0.5182	3.7058
2.88	0.5033	0.2333	0.4513	0.5169	3.7771
2.90	0.5065	0.2307	0.4474	0.5155	3.8498
2.92	0.5097	0.2281	0.4436	0.5142	3.9238
2.94	0.5129	0.2256	0.4398	0.5129	3.9993
2.96	0.5160	0.2231	0.4360	0.5116	4.0762
2.98	0.5191	0.2206	0.4323	0.5104	4.1547

M	$\bar{f}L^*/D$	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
3.00	0.5222	0.2182	0.4286	0.5092	4.2346
3.02	0.5252	0.2158	0.4249	0.5080	4.3160
3.04	0.5281	0.2135	0.4213	0.5068	4.3989
3.06	0.5310	0.2112	0.4177	0.5056	4.4835
3.08	0.5339	0.2090	0.4142	0.5045	4.5696
3.10	0.5368	0.2067	0.4107	0.5034	4.6573
3.12	0.5396	0.2045	0.4072	0.5023	4.7467
3.14	0.5424	0.2024	0.4038	0.5012	4.8377
3.16	0.5451	0.2002	0.4004	0.5001	4.9304
3.18	0.5478	0.1981	0.3970	0.4991	5.0248
3.20	0.5504	0.1961	0.3937	0.4980	5.1209
3.22	0.5531	0.1940	0.3904	0.4970	5.2189
3.24	0.5557	0.1920	0.3872	0.4960	5.3186
3.26	0.5582	0.1901	0.3839	0.4951	5.4201
3.28	0.5607	0.1881	0.3807	0.4941	5.5234
3.30	0.5632	0.1862	0.3776	0.4931	5.6286
3.32	0.5657	0.1843	0.3745	0.4922	5.7357
3.34	0.5681	0.1825	0.3714	0.4913	5.8448
3.36	0.5705	0.1806	0.3683	0.4904	5.9558
3.38	0.5729	0.1788	0.3653	0.4895	6.0687
3.40	0.5752	0.1770	0.3623	0.4886	6.1837
3.42	0.5775	0.1753	0.3594	0.4878	6.3007
3.44	0.5798	0.1736	0.3564	0.4869	6.4197
3.46	0.5820	0.1718	0.3535	0.4861	6.5409
3.48	0.5842	0.1702	0.3507	0.4853	6.6642
3.50	0.5864	0.1685	0.3478	0.4845	6.7896
3.52	0.5886	0.1669	0.3450	0.4837	6.9172
3.54	0.5907	0.1653	0.3422	0.4829	7.0470
3.56	0.5928	0.1637	0.3395	0.4821	7.1791
3.58	0.5949	0.1621	0.3368	0.4813	7.3134
3.60	0.5970	0.1606	0.3341	0.4806	7.4501
3.62	0.5990	0.1590	0.3314	0.4799	7.5891
3.64	0.6010	0.1575	0.3288	0.4791	7.7304
3.66	0.6030	0.1560	0.3262	0.4784	7.8742
3.68	0.6049	0.1546	0.3236	0.4777	8.0204
3.70	0.6068	0.1531	0.3210	0.4770	8.1690
3.72	0.6087	0.1517	0.3185	0.4763	8.3202
3.74	0.6106	0.1503	0.3160	0.4757	8.4739
3.76	0.6125	0.1489	0.3135	0.4750	8.6302
3.78	0.6143	0.1475	0.3111	0.4743	8.7891
3.80	0.6161	0.1462	0.3086	0.4737	8.9506
3.82	0.6179	0.1449	0.3062	0.4730	9.1147
3.84	0.6197	0.1436	0.3039	0.4724	9.2816
3.86	0.6214	0.1423	0.3015	0.4718	9.4513
3.88	0.6231	0.1410	0.2992	0.4712	9.6237
3.90	0.6248	0.1397	0.2969	0.4706	9.7989
3.92	0.6265	0.1385	0.2946	0.4700	9.9770
3.94	0.6282	0.1372	0.2923	0.4694	10.1580
3.96	0.6298	0.1360	0.2901	0.4688	10.3419
3.98	0.6315	0.1348	0.2879	0.4683	10.5288
4.00	0.6331	0.1336	0.2857	0.4677	10.7187
∞	0.8215	0	0	0.4082	∞

TABLE III : Non-viscous compressible flow with heat transfer in a constant cross-section pipeline, ideal gas (γ = 1.4) (Rayleigh curves).

M	T_0/T_0^*	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
0.00	0.0000	2.4000	0.0000	∞	1.2679
0.02	0.0019	2.3987	0.0023	1042.2500	1.2675
0.04	0.0076	2.3946	0.0092	261.0000	1.2665
0.06	0.0171	2.3880	0.0205	116.3241	1.2647
0.08	0.0302	2.3787	0.0362	65.6875	1.2623
0.10	0.0468	2.3669	0.0560	42.2500	1.2591
0.12	0.0666	2.3526	0.0797	29.5185	1.2554
0.14	0.0895	2.3359	0.1069	21.8418	1.2510
0.16	0.1151	2.3170	0.1374	16.8594	1.2461
0.18	0.1432	2.2959	0.1708	13.4434	1.2406
0.20	0.1736	2.2727	0.2066	11.0000	1.2346
0.22	0.2057	2.2477	0.2445	9.1922	1.2281
0.24	0.2395	2.2209	0.2841	7.8171	1.2213
0.26	0.2745	2.1925	0.3250	6.7470	1.2140
0.28	0.3104	2.1626	0.3667	5.8980	1.2064
0.30	0.3469	2.1314	0.4089	5.2130	1.1985
0.32	0.3837	2.0991	0.4512	4.6523	1.1904
0.34	0.4206	2.0657	0.4933	4.1877	1.1822
0.36	0.4572	2.0314	0.5348	3.7984	1.1737
0.38	0.4935	1.9964	0.5755	3.4688	1.1652
0.40	0.5290	1.9608	0.6151	3.1875	1.1566
0.42	0.5638	1.9247	0.6535	2.9454	1.1480
0.44	0.5975	1.8882	0.6903	2.7355	1.1394
0.46	0.6301	1.8515	0.7254	2.5525	1.1308
0.48	0.6614	1.8147	0.7587	2.3918	1.1224
0.50	0.6914	1.7778	0.7901	2.2500	1.1141
0.52	0.7199	1.7409	0.8196	2.1243	1.1059
0.54	0.7470	1.7043	0.8469	2.0122	1.0979
0.56	0.7725	1.6678	0.8723	1.9120	1.0901
0.58	0.7965	1.6316	0.8955	1.8219	1.0826
0.60	0.8189	1.5957	0.9167	1.7407	1.0753
0.62	0.8398	1.5603	0.9358	1.6673	1.0682
0.64	0.8592	1.5253	0.9530	1.6006	1.0615
0.66	0.8771	1.4908	0.9682	1.5399	1.0550
0.68	0.8935	1.4569	0.9814	1.4844	1.0489
0.70	0.9085	1.4235	0.9929	1.4337	1.0431
0.72	0.9221	1.3907	1.0026	1.3871	1.0376
0.74	0.9344	1.3585	1.0106	1.3442	1.0325
0.76	0.9455	1.3270	1.0171	1.3047	1.0278
0.78	0.9553	1.2961	1.0220	1.2682	1.0234
0.80	0.9639	1.2658	1.0255	1.2344	1.0193
0.82	0.9715	1.2362	1.0276	1.2030	1.0157
0.84	0.9781	1.2073	1.0285	1.1738	1.0124

M	T_0/T_0^*	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
0.86	0.9836	1.1791	1.0283	1.1467	1.0095
0.88	0.9883	1.1515	1.0269	1.1214	1.0070
0.90	0.9921	1.1246	1.0245	1.0977	1.0049
0.92	0.9951	1.0984	1.0212	1.0756	1.0031
0.94	0.9973	1.0728	1.0170	1.0549	1.0017
0.96	0.9988	1.0479	1.0121	1.0354	1.0008
0.98	0.9997	1.0236	1.0064	1.0172	1.0002
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.02	0.9997	0.9770	0.9930	0.9838	1.0002
1.04	0.9989	0.9546	0.9855	0.9686	1.0008
1.06	0.9977	0.9327	0.9776	0.9542	1.0017
1.08	0.9960	0.9115	0.9691	0.9406	1.0031
1.10	0.9939	0.8909	0.9603	0.9277	1.0049
1.12	0.9915	0.8708	0.9512	0.9155	1.0070
1.14	0.9887	0.8512	0.9417	0.9039	1.0095
1.16	0.9856	0.8322	0.9320	0.8930	1.0124
1.18	0.9823	0.8137	0.9220	0.8826	1.0157
1.20	0.9787	0.7958	0.9118	0.8727	1.0194
1.22	0.9749	0.7783	0.9015	0.8633	1.0235
1.24	0.9709	0.7613	0.8911	0.8543	1.0279
1.26	0.9668	0.7447	0.8805	0.8458	1.0328
1.28	0.9624	0.7287	0.8699	0.8376	1.0380
1.30	0.9580	0.7130	0.8592	0.8299	1.0437
1.32	0.9534	0.6978	0.8484	0.8225	1.0497
1.34	0.9487	0.6830	0.8377	0.8154	1.0561
1.36	0.9440	0.6686	0.8269	0.8086	1.0629
1.38	0.9391	0.6546	0.8161	0.8021	1.0701
1.40	0.9343	0.6410	0.8054	0.7959	1.0777
1.42	0.9293	0.6278	0.7947	0.7900	1.0856
1.44	0.9243	0.6149	0.7840	0.7843	1.0940
1.46	0.9193	0.6024	0.7735	0.7788	1.1028
1.48	0.9143	0.5902	0.7629	0.7736	1.1120
1.50	0.9093	0.5783	0.7525	0.7685	1.1215
1.52	0.9042	0.5668	0.7422	0.7637	1.1315
1.54	0.8992	0.5555	0.7319	0.7590	1.1419
1.56	0.8942	0.5446	0.7217	0.7545	1.1527
1.58	0.8892	0.5339	0.7117	0.7502	1.1640
1.60	0.8842	0.5236	0.7017	0.7461	1.1756
1.62	0.8792	0.5135	0.6919	0.7421	1.1877
1.64	0.8743	0.5036	0.6822	0.7383	1.2002
1.66	0.8694	0.4940	0.6726	0.7345	1.2131
1.68	0.8645	0.4847	0.6631	0.7310	1.2264
1.70	0.8597	0.4756	0.6538	0.7275	1.2402
1.72	0.8549	0.4668	0.6445	0.7242	1.2545
1.74	0.8502	0.4581	0.6355	0.7210	1.2692
1.76	0.8455	0.4497	0.6265	0.7178	1.2843
1.78	0.8409	0.4415	0.6176	0.7148	1.2999
1.80	0.8363	0.4335	0.6089	0.7119	1.3159
1.82	0.8317	0.4257	0.6004	0.7091	1.3324
1.84	0.8273	0.4181	0.5919	0.7064	1.3494
1.86	0.8228	0.4107	0.5836	0.7038	1.3669
1.88	0.8185	0.4035	0.5754	0.7012	1.3849
1.90	0.8141	0.3964	0.5673	0.6988	1.4033

M	T_0/T_0^*	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
1.92	0.8099	0.3896	0.5594	0.6964	1.4222
1.94	0.8057	0.3828	0.5516	0.6940	1.4417
1.96	0.8015	0.3763	0.5439	0.6918	1.4616
1.98	0.7974	0.3699	0.5364	0.6896	1.4821
2.00	0.7934	0.3636	0.5289	0.6875	1.5031
2.02	0.7894	0.3575	0.5216	0.6854	1.5246
2.04	0.7855	0.3516	0.5144	0.6835	1.5467
2.06	0.7816	0.3458	0.5074	0.6815	1.5693
2.08	0.7778	0.3401	0.5004	0.6796	1.5924
2.10	0.7741	0.3345	0.4936	0.6778	1.6162
2.12	0.7704	0.3291	0.4868	0.6760	1.6404
2.14	0.7667	0.3238	0.4802	0.6743	1.6653
2.16	0.7631	0.3186	0.4737	0.6726	1.6908
2.18	0.7596	0.3136	0.4673	0.6710	1.7168
2.20	0.7561	0.3086	0.4611	0.6694	1.7434
2.22	0.7527	0.3038	0.4549	0.6679	1.7707
2.24	0.7493	0.2991	0.4488	0.6664	1.7986
2.26	0.7460	0.2945	0.4428	0.6649	1.8271
2.28	0.7428	0.2899	0.4370	0.6635	1.8562
2.30	0.7395	0.2855	0.4312	0.6621	1.8860
2.32	0.7364	0.2812	0.4256	0.6607	1.9165
2.34	0.7333	0.2769	0.4200	0.6594	1.9476
2.36	0.7302	0.2728	0.4145	0.6581	1.9794
2.38	0.7272	0.2688	0.4091	0.6569	2.0119
2.40	0.7242	0.2648	0.4038	0.6557	2.0451
2.42	0.7213	0.2609	0.3986	0.6545	2.0789
2.44	0.7184	0.2571	0.3935	0.6533	2.1136
2.46	0.7156	0.2534	0.3885	0.6522	2.1489
2.48	0.7128	0.2497	0.3836	0.6511	2.1850
2.50	0.7101	0.2462	0.3787	0.6500	2.2218
2.52	0.7074	0.2427	0.3739	0.6489	2.2594
2.54	0.7047	0.2392	0.3692	0.6479	2.2978
2.56	0.7021	0.2359	0.3646	0.6469	2.3370
2.58	0.6995	0.2326	0.3601	0.6459	2.3770
2.60	0.6970	0.2294	0.3556	0.6450	2.4177
2.62	0.6945	0.2262	0.3512	0.6440	2.4593
2.64	0.6921	0.2231	0.3469	0.6431	2.5018
2.66	0.6896	0.2201	0.3427	0.6422	2.5451
2.68	0.6873	0.2171	0.3385	0.6413	2.5892
2.70	0.6849	0.2142	0.3344	0.6405	2.6343
2.72	0.6826	0.2113	0.3304	0.6397	2.6802
2.74	0.6804	0.2085	0.3264	0.6388	2.7270
2.76	0.6781	0.2058	0.3225	0.6380	2.7748
2.78	0.6760	0.2031	0.3186	0.6372	2.8235
2.80	0.6738	0.2004	0.3149	0.6365	2.8731
2.82	0.6717	0.1978	0.3111	0.6357	2.9237
2.84	0.6696	0.1953	0.3075	0.6350	2.9752
2.86	0.6675	0.1927	0.3039	0.6343	3.0277
2.88	0.6655	0.1903	0.3004	0.6336	3.0813
2.90	0.6635	0.1879	0.2969	0.6329	3.1358
2.92	0.6615	0.1855	0.2934	0.6322	3.1914
2.94	0.6596	0.1832	0.2901	0.6315	3.2481
2.96	0.6577	0.1809	0.2868	0.6309	3.3058

M	T_0/T_0^*	p/p^*	T/T^*	ρ/ρ^* V^*/V	p_0/p_0^*
2.98	0.6558	0.1787	0.2835	0.6303	3.3646
3.00	0.6540	0.1765	0.2803	0.6296	3.4244
3.02	0.6522	0.1743	0.2771	0.6290	3.4854
3.04	0.6504	0.1722	0.2740	0.6284	3.5476
3.06	0.6486	0.1701	0.2709	0.6278	3.6108
3.08	0.6469	0.1681	0.2679	0.6273	3.6752
3.10	0.6452	0.1660	0.2650	0.6267	3.7408
3.12	0.6435	0.1641	0.2620	0.6261	3.8076
3.14	0.6418	0.1621	0.2592	0.6256	3.8756
3.16	0.6402	0.1602	0.2563	0.6251	3.9449
3.18	0.6386	0.1583	0.2535	0.6245	4.0154
3.20	0.6370	0.1565	0.2508	0.6240	4.0871
3.22	0.6354	0.1547	0.2481	0.6235	4.1601
3.24	0.6339	0.1529	0.2454	0.6230	4.2345
3.26	0.6324	0.1511	0.2428	0.6225	4.3101
3.28	0.6309	0.1494	0.2402	0.6221	4.3871
3.30	0.6294	0.1477	0.2377	0.6216	4.4655
3.32	0.6280	0.1461	0.2352	0.6211	4.5452
3.34	0.6265	0.1444	0.2327	0.6207	4.6263
3.36	0.6251	0.1428	0.2303	0.6202	4.7089
3.38	0.6237	0.1412	0.2279	0.6198	4.7929
3.40	0.6224	0.1397	0.2255	0.6194	4.8783
3.42	0.6210	0.1381	0.2232	0.6190	4.9652
3.44	0.6197	0.1366	0.2209	0.6185	5.0536
3.46	0.6184	0.1351	0.2186	0.6181	5.1435
3.48	0.6171	0.1337	0.2164	0.6177	5.2350
3.50	0.6158	0.1322	0.2142	0.6173	5.3280
3.52	0.6145	0.1308	0.2120	0.6170	5.4226
3.54	0.6133	0.1294	0.2099	0.6166	5.5188
3.56	0.6121	0.1280	0.2078	0.6162	5.6167
3.58	0.6109	0.1267	0.2057	0.6158	5.7161
3.60	0.6097	0.1254	0.2037	0.6155	5.8173
3.62	0.6085	0.1241	0.2017	0.6151	5.9201
3.64	0.6074	0.1228	0.1997	0.6148	6.0247
3.66	0.6062	0.1215	0.1977	0.6144	6.1310
3.68	0.6051	0.1202	0.1958	0.6141	6.2390
3.70	0.6040	0.1190	0.1939	0.6138	6.3488
3.72	0.6029	0.1178	0.1920	0.6134	6.4605
3.74	0.6018	0.1166	0.1902	0.6131	6.5739
3.76	0.6008	0.1154	0.1884	0.6128	6.6892
3.78	0.5997	0.1143	0.1866	0.6125	6.8064
3.80	0.5987	0.1131	0.1848	0.6122	6.9255
3.82	0.5977	0.1120	0.1830	0.6119	7.0466
3.84	0.5967	0.1109	0.1813	0.6116	7.1696
3.86	0.5957	0.1098	0.1796	0.6113	7.2945
3.88	0.5947	0.1087	0.1779	0.6110	7.4215
3.90	0.5937	0.1077	0.1763	0.6107	7.5505
3.92	0.5928	0.1066	0.1746	0.6104	7.6815
3.94	0.5918	0.1056	0.1730	0.6102	7.8147
3.96	0.5909	0.1046	0.1714	0.6099	7.9499
3.98	0.5900	0.1036	0.1699	0.6096	8.0873
4.00	0.5891	0.1026	0.1683	0.6094	8.2268
∞	0.4898	0	0	0.5833	∞

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