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Mild Solutions for Some Evolution Problems with State-Dependent Delays

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Abstract :

We are studying different classes of partial functional and neutral functional evolution equations which are perturbed and nonperturbed, with finite and infinite delay depending and non-depending on the solution and having a first order, second order, and even fractional derivative order between 0 and 1 in the sense of Caputo and involving local and nonlocal conditions of the following general form

$$\begin{cases} \frac{d^\alpha}{dt^\alpha} [y(t) - g(t, y_{\rho(t, y_t)})] = A(t)y(t) + Cu(t) + f(t, y_{\rho(t, y_t)}), & \text{a.e. } t \in J, \\ y(t) + h_t(y) = \phi(t), & t \in H \end{cases} \quad (1)$$

for given α , g , ρ , $A(\cdot)$, C , u , f , J , $h_t(\cdot)$, ϕ and H .

To obtain the existence and controllability of mild solutions, sufficient conditions will be considered in studying the different classes of *evolution* problems with state-dependent delays and nonlocal conditions. The used method is to transform the search for the existence of mild solutions to the search for the existence of fixed points of appropriate operators by applying different nonlinear alternatives to ensure the existence of the fixed points of this operator, which are the weak solutions to our problems. This method is based on famous fixed-point theorems combined with semigroup theory.

Keywords : Evolution equations; neutral problems; perturbed problems; second-order; fractional order derivative; mild solution; controllability; state-dependent delay; nonlocal conditions; fixed point; nonlinear alternative; semigroup theory.

Mathematics Subject Classification : 26A33; 34G20; 34G25; 34K26; 34K37; 34K40; 37L05; 74H20; 93B05.

References

- [1] I. Abibssi and S. Baghli-Bendimerad, On nonlocal second order perturbed pseudo integro-differential evolution equations with infinite state-dependent delay, *Mathematica Applicanda : Functional Analysis Applications*, Accepted (2024).
- [2] C. Boudefla and S. Baghli-Bendimerad, Nonlocal controllability of mild solutions for evolution equations with state-dependent delay in Fréchet spaces, *International Journal of Applied Mathematics*, **37** (1), 53-71, (2024)
- [3] C. Boudefla and S. Baghli-Bendimerad, Nonlocal controllability of mild solutions for neutral evolution equations with state-dependent delay in Fréchet spaces, *Arch. Control Sci.*, **34** LXX (1), 117–148, (2024)
- [4] C. Boudefla, F. Sahraoui and S. Baghli-Bendimerad, Controllability of Mild Solutions for Second-Order Neutral Evolution Equations with State-Dependent Delay, *Complex Anal. Oper. Theory* **18**, Article 80, 24pp, (2024)
- [5] N. Lachachi-Merad, S. Baghli-Bendimerad and M. Benchohra, Unique Mild Solutions for Caputo's Fractional Perturbed Evolution Equations with State-Dependent Delay, *Rocky Mountain Journal of Mathematics*, accepted, (2023)
- [6] N. Lachachi-Merad, S. Baghli-Bendimerad, M. Benchohra and D. Aouad, Unique Mild Solution for Fractional Partial and Neutral Evolution Equations with State-Dependent Delay, *Evolution Equations and Control Theory*, **13** (1), 160-172, (2024)
- [7] N. Lachachi-Merad, S. Baghli-Bendimerad, M. Benchohra and Erdal Karapinar, Nonlocal Partial Fractional Evolution Equations with State-Dependent Delay, *Proyecciones Journal of Mathematics*, (*Antofagasta, On line*), **42** (5), 191-1210, (2023)