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Model reduction of functional processes with unbounded variance

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Abstract

Model reduction seeks to replace a high-order dynamical system with a lower-dimensional surrogate that preserves the dominant features of the original system's response.

Established methodologies including Proper Orthogonal Decomposition (POD), Principal Component Analysis (PCA), and Karhunen-Loève (KL) decomposition are extensively employed for this purpose. Given the fundamental commonality underlying these approaches, this investigation focuses on PCA of functional processes $(X_t)_{t\in\mathbb{R}_+}$ (X_t belongs to some Hilbert space $\mathbb{H}$) using a spectral decomposition of bounded variance-covariance operator Γ associated to the process.

When this operator Γ becomes unbounded, specifically in the case where $X_t \in L^p$ and $1 < p < 2$, we establish a generalization of the KL expansion in L^p using a nonlinear homeomorphic transformation upon which the classical KL expansion is subsequently applied. We further characterize specific subsets of L^p wherein this transformation exhibits isometry-preserving properties.

To formalize this framework, consider the stochastic process

$$X : (\Omega \times \mathbb{T},\, \mathcal{A} \otimes \mathcal{T},\, P \times \mu) \longmapsto (\mathbb{R},\, \mathcal{B}_{\mathcal{R}}) \quad \text{where} \quad \mathbb{T} \subset \mathbb{R}_+$$

When either the measure P or the measure μ lacks a finite second moment, we provide an explicit construction of a nonlinear homeomorphic transformation that enables the application of the KL expansion to the transformed process.

An application to symmetric Lévy α-stable processes is presented to illustrate the efficacy of this non-linear methodology.

keywords : Covariance Operator, Functional data, Karhunen-Loève (KL) decomposition, Hilbert space, Principal Component Analysis, Reflexive Banach space.

Mathematics Subject Classification : 62R10, 60G20, 60G5x, 47A70, 46B10.

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