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Well-posedness and general decay estimates of solutions for a nonlinear system of coupled hyperbolic and parabolic equations

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Abstract

Coupled hyperbolic and parabolic systems have been widely studied in mathematical physics and engineering due to their applications in heat conduction, elasticity, viscoelasticity, and thermomechanical interactions. These models describe various physical phenomena, such as wave propagation in elastic media with thermal effects and fluid-structure interactions in porous materials. In particular, understanding the existence, uniqueness, and long-time behavior of solutions to such systems is crucial for predicting their stability and response to external influences.

In this work, we consider a nonlinear coupled system consisting of a wave equation and a heat equation defined in a bounded domain. First, we establish the well-posedness of solutions in both the degenerate and non-degenerate cases, by using the Faedo-Galerkin scheme. Then, we prove the exponential stability by constructing a suitable Lyapunov functional in the non-degenerate case. We show that the general stability estimates in the degenerate case. The proof is based on the multiplier method and general weighted integral inequalities proved by the second author in [4], and some properties of convex functions, in particular, the dual function of convex function to use the general Young and Jensen's inequalities These arguments of convexity were introduced and developed by Lasiecka et al. ([5], [6]), and used by Alabau-Boussouira [1]. Finally, we give many significant examples to illustrate how to derive from our general estimates the polynomial, exponential or logarithmic decay.

Keywords: Coupled system, degenerate or nondegenerate Kirchhoff term, well-posedness, multiplier method, integral inequalities, exponential stability, general stability.

Mathematics Subject Classification. Primary: 35B40, Secondary, 35K05, 35L05.

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