

## How to apply Tikhonov's convergence Theorem for slow-fast systems to an age structured epidemiological model

Karim Yadi<sup>1</sup> & Radhwane Benkhaled<sup>2</sup>

Université Aboubekr Belkaïd Tlemcen

<sup>1</sup>Laboratoire Systèmes Dynamiques et Applications

<sup>2</sup>Laboratoire Analyse Non Linéaire et Mathématiques Appliquées

karim.yadi@univ-tlemcen.dz / yadikdz@yahoo.fr

### Abstract

We consider a simple SIS model with a general nonlinear treatment function. We combine it with a demographic model keeping in mind the difference in time scale. A slow-fast system emerges. Our purpose is to show how Tikhonov's Theorem for singularly perturbed systems can be applied to reduce the whole model to planar systems that approximate its slow phases. Based on assumptions made about the treatment function, we determine graphically the number and the attractivity of slow surfaces on which reduced problems are defined. Approximation results are given on bounded, arbitrarily large and unbounded time intervals. A specific example is examined for a function that combines a natural recovery rate and a homographic treatment function that takes into account the limitations of hospital capacities. Numerical illustration are discussed in this case and some epidemiological explanations are given. The starting point for this work is the article of Banasiak *et al.* [1].

**Mathematics Subject Classification :** 34E15, 92D30.

**Keywords:** Epidemiological models, demographic models, singular perturbations, Tikhonov's theorem, slow manifold, Reduced problems, Treatment function.

### References

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