Penalty Sponge Layers for the Anisotropic Advection-Diffusion Equation

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Abstract

We propose a new domain truncation technique, the *Penalty Sponge Layer (PSL)*, for the numerical solution of the two-dimensional anisotropic advection-diffusion equation in unbounded domains. Inspired by recent advances in hyperbolic systems, the PSL naturally damps diffusion within the artificial layer while shifting the damping rate into the advection velocity, effectively transitioning from a diffusion-dominated to an advection-dominated regime. Key advantages of the method include its ability to handle corner layers, along with well-posedness and stability properties that are directly inherited from the original problem. Numerical validation is performed using both reference and exact solutions, confirming the efficiency and accuracy of the proposed model.

Mathematics Subject Classification : 35L05, 35L45, 35B35.

Keywords: Advection-diffusion, Penalty Sponge Layer, Well-posedness, Stability, Energy decay .

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