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Mild and Integral Solutions for Semilinear Differential Equations on the Half Line by Diagonalization Method

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Abstract

We give some existence of bounded mild and integral solutions for semlinear differential equations. We consider the cases when the linear part of the equation generates a C_0 semigroup as well as an integrated semigroup. An application is made of a Darbo fixed point theorem given in [8] associated with the diagonalization method and the concept of measure of noncompactness. The present results initiate the application of such method to semilinar differential equations on the half line.

We deal with the class of first order semilinear differential equations of the following form:

$$\begin{cases} u'(t) = Au(t) + f(t, u(t)); & \text{if } t \in \mathbb{R}_+; \\ u(0) = u_0, \ u \text{ is bounded on } \mathbb{R}_+, \end{cases} \tag{1}$$

where $f : \mathbb{R}_+ \times E \to E$ is a given function, u_0 is a given constant, $(E, \|\cdot\|)$ is a real (or complex) Banach space and $A : D(A) \subset E \to E$ is the infinitesimal generator of a C_0-semigroup $\{T(t)\}_{t\geq 0}$.

Next, we shall be concerned with existence of integral mild solutions for problem (1), in the case where $A : D(A) \subset E \to E$ is a nondensely defined closed linear operator on the Banach space E.

Significant developments in evolution equations and inclusions with delays was developed during these last years for first order as in [2]-[10] when A depends on t. First order integrodifferential equations with infinite delay was considered by Meslem *et al.* in [12] and integrodifferential inclusions with non instantaneous impulses in [13]. Based on the papers of Benchohra *et al.* [1, 6, 7], this work initiates the application of the diagonalization method to first order semilinear differential equations in case that the linear part of the equation generates a C_0-semigroup as well as an integrated semigroup [11].

Mathematics Subject Classification : 34G20; 34A37.

Keywords: Integro-differential equation; impulsive; resolvent operator; measure of noncompactness; periodic mild solution; Poincaré operator; fixed point.

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