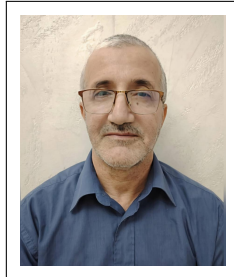


**Ulam-Hyers-Aoki-Rassias Stability:  
Approximation, Robustness and Persistence of Structures**



**Jamal Eddine Lazreg**

Laboratory of Mathematics (LDM)  
Djillali Liabes University of Sidi Bel Abbès  
lazregjamal@yahoo.fr

**Abstract**

This presentation is devoted to the study of Ulam-Hyers-Aoki-Rassias stability, highlighting its historical development, conceptual foundations, and contemporary advances. We begin by recalling the pioneering intuition introduced by George Pólya and Gábor Szegő in 1925, which suggests that a mathematical relation that is approximately satisfied may reveal an underlying exact structure. This idea was later formalized by Stanislaw Ulam in 1940 and subsequently solved by Donald Hyers in 1941, giving rise to the notion of stability of functional equations. We then introduce the fundamental concepts of error, approximation, and robustness, showing how a mathematical structure can persist under perturbations. This approach naturally leads to modern generalizations, particularly the results of Aoki-Rassias, as well as extreme phenomena such as superstability and hyperstability. Finally, we discuss current developments and perspectives of this theory, especially its connections with functional analysis, operator theory, group theory, and its potential applications in modeling and artificial intelligence. Overall, this study highlights that Ulam stability is not merely a technical result, but a genuine theory of robustness of mathematical structures.

**Keywords and phrases:** Ulam-Hyers-Aoki-Rassias stability, approximation, robustness, functional equations, superstability, hyperstability, mathematical structures, operators, rigidity.

**References**

- [1] Janusz Brzdek, Dorian Popa, Themistocles M. Rassias. *Ulam Type Stability*, Springer, 2019.
- [2] Janusz Brzdek, Dorian Popa, Ioan Rasa, Bing Xu. *Ulam Stability of operators*, Academic Press, 2018.
- [3] S.-M. Jung. *Hyers-Ulam-Rassias Stability of Functional Equations in Nonlinear Analysis*, Springer, New York, 2011.