A Scientific Day on Mathematics and Computer Science

Mathematics and Computer Science Seminar

dedicated to the retirement of

Dr. Boudekhil CHAFI

July 09, 2026

Laboratory of Mathematics (LDM)

Faculty of Exact Sciences

Djillali Liabès University of Sidi Bel Abbès

Program overview

08:30 - 09:00 Opening ceremony and homages

09:00 - 09:40 Conference/debate
The Role of Kuratowski's Functional in the Genesis of the Measure of Noncompactness
Pr. Abdelghani OUAHAB
University of Sidi Bel Abbès

09:40 - 10:20 Conference/debate
On an extension of the simplified SWU mapping in constant time
Pr. Kamel Mohamed FARAOUN
University of Sidi Bel Abbès

10:20 - 11:00 Conference/debate
From Symmetric to Convex Fractional Differentiation
Pr. Maamar BENBACHIR
NHSM, Sidi Abdellah, Algiers

11:00 - 11:40 Coffee break / Posters (Ph.D students)

11:40 - 12:20 Conference/debate
Stabilization of a wave equation with a general internal feedback of diffusive type
Pr. Abbès BENAISSA
University of Sidi Bel Abbès

12:20 - 13:00 Conference/debate
The KV-cohomology of flat Poisson manifolds
Dr. Ahmed ZEGLAOUI
NHSAST, Sidi Abdellah, Algiers

13:00 - 13:20 Speech by Dr. Boudekhil CHAFI

13:20 Closing ceremony.

Contents

Introduction

This volume collects the abstracts of the talks and posters presented at the Scientific Day - dedicated to the retirement of the colleague Dr. Chafi Boudekhil - organized by the Laboratory of Mathematics (LDM) of Djillali Liabès University of Sidi Bel Abbès, held on July 09, 2026, at Hotel Eden, Sidi Bel Abbès.

The seminar brought together researchers from various fields of mathematics and from different academic institutions

We thank all speakers, presenters, organizers and participants for their contributions.

The Organizing Committee
July 2026

Part I

Conferences (Abstracts)

1. The Role of Kuratowski's Functional in the Genesis of the Measure of Noncompactness

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Abstract

The concept of the measure of noncompactness occupies a central place in modern functional analysis, serving as a powerful instrument in fixed point theory, the study of differential and integral equations, operator theory, and approximation theory. At the historical foundation of this theory stands the seminal contribution of Kazimierz Kuratowski [7], whose 1930 definition of what is now known as the *Kuratowski measure of noncompactness* (denoted by α) was used to give different generalizations of Cantor's theorem.

In 1955, Darbo [5] established fundamental properties of the functional α. His results led to a significant generalization of the Schauder fixed point theorem and provided new tools for addressing the Peano existence problem in infinite-dimensional normed spaces. In 1965, Goldenshtein and Markus [6] introduced the term *measure of noncompactness*, which they called the Hausdorff measure of noncompactness.

In 1967, Sadovskii [9] introduced the concept of *condensing mappings* via the measure of noncompactness to extend Schauder and Leray–Schauder degree theory and the solvability of nonlinear operator equations. The first abstract definition of the measure of noncompactness was introduced in 1968 by Sadovskii [10]; a second axiomatic definition based on properties of the measure of noncompactness of Kazimierz Kuratowski was introduced by Nussbaum [8] for the study of some classes of functional periodic differential equations, and in 1980 J. Banas and K. Goebel [2] introduced a new axiomatic definition of noncompactness in Banach spaces.

In 1977, De Blasi [4] introduced the notion of the *measure of weak noncompactness*.

Also in this talk we use the axiomatic definition of MNCs in metric spaces, applying it to construct a functional in some probability space. Finally, we give the concept of the measure of noncompactness in Gromov–Hausdorff space.

Mathematics Subject Classification. 47H08, 47H10; Secondary 46B50, 54E52.

Keywords and phrases. Kuratowski measure of noncompactness, functional, Hausdorff measure, condensing mappings, fixed point theory, probability space, Gromov–Hausdorff space.

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2. On an extension of the simplified SWU mapping in constant time

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Abstract

We presents a novel advancements in extending the simplified Shallue–van de Woestijne–Ulas (SWU) mapping technique to BLS24 and BLS48 elliptic curves. The SWU method ensures constant-time execution for point mapping on these curves, effectively mitigating side-channel vulnerabilities and addressing limitations in existing implementations. An innovative formalism is introduced to enable the efficient computation of isogeny maps for curve twists over corresponding high-order extension fields, supported by rigorous mathematical justifications and proofs. The proposed method is evaluated on several widely used BLS24 and BLS48 curves, with the computed isogenies benchmarked through an optimized Rust implementation. Additionally, an enhanced variant of the simplified SWU mapping is presented, eliminating the costly inversion operation and thereby improving overall performance. Finally, four optimized constructions of BLS48 curves are proposed, enhancing their suitability for security applications at both 256-bit and 128-bit levels.

Mathematics Subject Classification. 11G20, 11T71, 14G50, 11Y16.

Keywords and phrases. SWU Mapping, Pairing-based cryptography, BLS Curves, Constant-time implementation.

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3. From Symmetric to Convex Fractional Differentiation

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Abstract

We introduce a novel class of fractional differential operators that interpolates continuously between the classical left-sided and right-sided Caputo derivatives through a scalar weighting parameter $\lambda \in [0,1]$. The construction rests on a convex combination of the left and right Riemann–Liouville integral operators applied prior to integer-order differentiation, yielding what we term the *convex Caputo derivative* D_B^α. This unified framework recovers, as distinguished special cases, the standard left Caputo derivative when $\lambda = 1$, the right Caputo derivative when $\lambda = 0$, and the well-known symmetric Riesz–Caputo derivative when $\lambda = \frac{1}{2}$, while the intermediate regime $\lambda \neq \frac{1}{2}$ produces a genuinely asymmetric operator with no classical counterpart.

For this family of operators, we develop a systematic operational calculus: explicit composition identities with fractional integrals are established, and a complete proof of the fundamental theorem of fractional calculus is given in this extended setting. Turning to differential equations governed by the new operator, we formulate a natural initial value problem and prove, via a Banach contraction mapping argument.

Keywords and phrases. Fractional calculus; Convex fractional derivative; Riesz–Caputo operator; Weighted fractional operators; Existence and uniqueness; Fixed-point methods.

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4. Stabiliation of a Wave Equation with a General Internal Feedback of Diffusive Type

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Abstract

In this work, we study well-posedness and asymptotic stability of a wave equation with a general internal control condition of diffusive type.We prove that the system lacks exponential stability. Furthermore, we show an explicit and general decay rate result, using the semigroup theory of linear operators and an estimate on the resolvent of the generator associated with the semigroup.

Mathematics Subject Classification. 35B40, 47D03, 74D05, 93D15.

Keywords and phrases. General internal dissipation of diffusive type, General decay rate, Wave equation.

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5. On the KV-cohomology of flat Poisson manifolds

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Abstract

Hochschild cohomology and Chevalley–Eilenberg cohomology control, among other things, deformations and extensions of associative algebras and Lie algebras, respectively. For an algebra that is neither associative nor a Lie algebra, defining a module structure is generally easy, but defining cohomology theories is not at all obvious. The case of Koszul–Vinberg algebras perfectly illustrates this difficulty. In 1968, A. Nijenhuis gave a first definition of the cohomology of a KV-algebra, using the Chevalley–Eilenberg cohomology of the Lie algebra defined by the commutator of the KV-product. An intrinsic definition was later given by M. Nguiffo Boyom in [6].

In this contribution, we introduce the notion of a locally flat Poisson manifold, together with its linear counterpart: the notion of a flat Lie algebra, by analogy with the covariant case of locally flat manifolds.

To each locally flat Poisson manifold, we associate a Koszul–Vinberg algebra, which defines a left Koszul–Vinberg module structure on the space of smooth functions on the underlying manifold. This allows us to introduce the Koszul–Vinberg cohomology of a locally flat Poisson manifold. We also investigate the linear version, for which we establish a link with the Chevalley–Eilenberg cohomology of the underlying Lie algebra.

Mathematics Subject Classification. Primary 53D05; Secondary 17D99; 17B30.

Keywords and phrases. Koszul–Vinberg cohomology, Locally flat Poisson manifolds, Flat Lie algebras, Chevalley–Eilenberg cohomology.

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Part II

Posters (Abstracts)

1. The Convergence Rate of the Hazard Function with Functional Explanatory Variable: Case of Spatial Data with the k-Nearest Neighbor Method

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Abstract

This work addresses the nonparametric estimation of the conditional hazard function when the explanatory variable is functional and the observations exhibit a spatial dependence structure. We propose a new estimator combining the k-nearest neighbors (kNN) procedure with the spatial functional approach, allowing for an adaptive, data-driven choice of the smoothing parameter, treated as a random variable. Under standard regularity conditions (small-ball conditions, spatial mixing, Hölder conditions on the hazard function), we establish the uniform almost-complete consistency of the estimator, with an explicit convergence rate. This result is obtained from intermediate results on the estimation of the conditional density and distribution function. An application to the estimation of the maximum risk point is then proposed. A Monte Carlo simulation study on a 30×30 spatial grid shows that the kNN estimator exhibits better robustness than the classical estimator in the presence of contaminated data (15% outliers). Finally, an application to real climatic data (125 NOAA stations, USA, 2000–2010) confirms the superiority of the kNN approach in terms of mean squared error (MSE = 0.21 versus 0.36 for the classical estimator).

Mathematics Subject Classification. 62G05; 62G20; 62M30 ; 62G08; 62H11.

Keywords and phrases. Hazard function; functional data; spatial data; k-nearest neighbors (kNN); uniform almost-complete consistency; spatial mixing processes; nonparametric estimation; risk point.

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2. From L'Hôpital's Question to Hilfer's Generalized Derivative

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Abstract

This work presents a comprehensive overview of the historical evolution and modern advancements in fractional calculus, tracing its journey from Guillaume de l'Hôpital's foundational question in 1695 to the development of contemporary generalized operators. We highlight the transition from classical memory-dependent formulations, such as Riemann-Liouville and Caputo derivatives, to the unified framework of the Hilfer fractional derivative. By introducing a type parameter $\beta \in [0,1]$, the Hilfer operator acts as an interpolating bridge between classical formulations. Furthermore, we explore recent generalizations, including the ψ-Hilfer and Hilfer-Hadamard derivatives, and discuss their emerging applications in modeling real-world complex systems, such as high-frequency electrical networks, anomalous calcium diffusion in neuro-biomedicine, and linearized fluid dynamics.

Mathematics Subject Classification. 26A33, 34A08, 01A05.

Keywords and phrases. Fractional Calculus, Hilfer Fractional Derivative, Historical Evolution, Memory Effects, Generalized Operators, Real-World Applications.

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3. From Dense to Nondense Domains: A Comparative Study of Partial Functional Integrodifferential Equations

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Abstract

This poster presents a comparative study of partial functional integro-differential equations in both dense and nondense domain settings. The objective is to highlight the mathematical differences between the classical resolvent operator approach and the integrated resolvent operator framework. In the dense domain case, the operator generates a C_0-semigroup, leading to the notion of mild solutions, whereas in the nondense domain case, the integrated semigroup theory provides the appropriate framework for integral solutions. The assumptions required in each setting, together with the corresponding existence and uniqueness results, are compared. Representative examples are included to illustrate both approaches. This comparative study provides a unified perspective on the analysis of functional integro-differential equations and opens the way to future extensions involving more general nonlinearities and fixed point techniques.

Mathematics Subject Classification. 34G20, 34K30, 47D06, 45J05, 47H10.

Keywords and phrases. Partial functional integro-differential equations; resolvent operator; integrated resolvent operator; existence of solutions; mild solution; integral solution; nondensely defined operator.

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