

Topics in Metric Fixed Point Theory

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Abstract

The metric fixed point theory has evolved from the famous Banach contraction principle (1922). This fundamental and powerful theorem has turned out to be an efficient tool with regards to the solvability of several linear and nonlinear equations. It has further a wide spectrum of applications in some disciplines of applied mathematics and economics. During the last five decades, a special attention has been given by researchers to metric spaces endowed with a graph structure, including partially ordered sets. In this respect, we will present most recent results and then raise some open problems.

Keywords and phrases: Fixed point theorem, Existence, Uniqueness, Approximation, Banach-Picard iteration, Ordered set, Graph structure.

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