

**Codazzi, Parallel and totally umbilical hypersurfaces
of some four-dimensional Thurston geometries**

Mohamed Belkhelfa

Faculty of Exact Sciences

Mustapha Stambouli University of Mascara, Algeria

belkhelfa@univ-mascara.dz, mohamed.belkhelfa@gmail.com

Abstract

In this talk we give the classification of Codazzi hypersurfaces, totally geodesic, parallel and totally umbilical hypersurfaces in some four-dimensional Thurston geometries namely in Nil^4 , $Sol_{m,n}^4$ and $Nil^3 \times \mathbb{R}$.

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